IMM - DTU

02407 Stochastic Processes
2014-9-15
DAME/BFN

Solution for exercise 3.1.1 in Pinsky and Karlin

The variable X_n are defined to be the number of diseased persons at the end of period n. From the conditinally indepence these variables is seen to form a Markov chain. From the assumptions the number of diseased persons can increase with at most one and can never decrease. To get a new person infected there needs to be one infected and one healthy person in the pair, and transmission has to happen. The probability of having one of each type in the pair is given by

$$\frac{\binom{X_n}{1}\binom{5-X_n}{1}}{\binom{5}{2}},$$

from which we derive

$$\mathbb{P}(X_{n+1} = X_n + 1) = \frac{\binom{X_n}{1}\binom{5-X_n}{1}}{\binom{5}{2}} \frac{1}{10}, \qquad 1 \le X_n \le 4.$$

In conclusion we get

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{24}{25} & \frac{1}{25} & 0 & 0 & 0 \\ 0 & 0 & \frac{47}{50} & \frac{3}{50} & 0 & 0 \\ 0 & 0 & 0 & \frac{47}{50} & \frac{3}{50} & 0 \\ 0 & 0 & 0 & 0 & \frac{24}{25} & \frac{1}{25} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$