IMM - DTU

04141 Stochastic Processes
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BFN/bfn

Exercise 18 (2/1/95 ex.2)

Consider the arrival of request to an inventory control system. The arrival process of requests can be described by a renewal process with the time interval between requests described by a distrbution with density f(t). The mean and variance of this distribution is μ and σ^2.

At a specific time unit the time until the next event is given by a distribution with density

$$v(t) = \frac{\int_t^\infty f(x)dx}{\mu}$$

Question 1

What is the expected number of requests during the next T time units.

The handling time of a request can be described by a distribution with density g(t)=$\eta e^{-\eta t}$. It can be assumed that requests is handled one at a time other requests will have to wait.

Question 2

Show that the probability that i requests can be handled in the time between two consecutive arrivals of requests is given by

$$a_i = \int_0^\infty \frac{(\eta t)^i}{i!} e^{-\eta t} f(t)dt$$

The next thing to consider is the number of requests in the system at the arrival of a new request. Let θ_i be the probability of finding i requests in the system at the arrival of a new request.

Question 3

Deduce a system of equation for the determination of $\vec{\theta}$ under stationarity.

We will now consider the time from the arrival of the n'th request until handling of that request.

Question 4

Show that the process describing the above mentioned waiting time is a random walk with continuous state space, parameterize the process and give distribution, mean and variance of the increments.

It can be assumed that

$$f(t) = \frac{3}{4}\eta\left(\frac{3}{4}\eta t\right)e^{-\frac{3}{4}\eta t}$$

Question 5

Present an explicit expression of the waiting time distribution.