IMM - DTU

02407 Stochastic Processes
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BFN/bfn

Exercise 33 (partly from R. W. Wolff)

Suppose the following weather types are possible in Napa Valley, an important wine site in California.

- Cool and clear
- Cool and foggy
- Hot and clear
- Rain

Suppose further that the weather type day n can be described in the following way:

- The probability of a cool and clear day will be
  - 0.2 if the weather type of day $n-1$ was cool and clear
  - 0.3 if the weather type of day $n-1$ was cool and foggy
  - 0.1 if the weather type of day $n-1$ was hot and clear
  - 0.4 if the weather type of day $n-1$ was rain
- The probability of a cool and foggy day will be
  - 0.4 if the weather type of day $n-1$ was cool and clear
  - 0.4 if the weather type of day $n-1$ was cool and foggy
  - 0.6 if the weather type of day $n-1$ was hot and clear
  - 0.2 if the weather type of day $n-1$ was rain
- The probability of a hot and clear day will be
  - 0.3 if the weather type of day $n-1$ was cool and clear
  - 0.1 if the weather type of day $n-1$ was cool and foggy
  - 0.2 if the weather type of day $n-1$ was hot and clear
  - 0.1 if the weather type of day $n-1$ was rain
- The probability of a rain day will be
  - 0.1 if the weather type of day $n-1$ was cool and clear
  - 0.2 if the weather type of day $n-1$ was cool and foggy
  - 0.1 if the weather type of day $n-1$ was hot and clear
  - 0.3 if the weather type of day $n-1$ was rain

Feel free to use matlab or any other computer tool available in any of the following questions if you find it convenient.

Question 1

Formulate a mathematical model describing the sequence of weather days.

Question 2

Characterise your model.

Question 3

Once having a rain day, what is the probability that we will have 6 further rain days in a row.

Question 4

What is the expected length of a sequence of cool and foggy days.

In the following we will assume that the weather type a certain day is cool and foggy.

Question 5

Calculate the probability that the weather type will be hot and clear in 5 days.

Question 6

Make a number of different realisations of the chain (20 days).

Question 7

Calculate the probability that a day some time in the far future is a rain day.

Question 8

Calculate the probability that the first rain day will occur in day 10.

Question 9

What is the expected number of rain days between two hot and clear days.

It is now cool and clear in the Napa Valley; the grapes are not quite ripe. Another hot spell would ripen them, but rain before a hot spell will ruin them. The grower must decide whether to pick less than ideal grapes now or gamble on the weather.

Question 10

Compute the probability that a hot spell will occur before rain.