

Exercise 6

Markov renewal processes

We consider a Markov renewal process with the kernel $\tilde{\mathbf{A}}(s)$, given by:

$$\tilde{\mathbf{A}}(s) = \begin{bmatrix} \frac{3}{s+5} & \frac{0.8}{s+2} \\ \frac{0.6}{s+3} + \frac{1.5}{s+5} & \frac{2}{(s+2)^2} \end{bmatrix}$$

X_n denotes the type of the n 'th event and τ_n denotes the time interval between the $(n-1)$ 'th and the n 'th event.

Question 1 Find the probabilities $p_{ij} = \text{Prob}\{X_{n+1} = j | X_n = i\}$, $i, j = 1, 2$.

Question 2 Find the conditional distributions
 $H_{ij}(t) = \text{Prob}\{\tau_n \leq t | X_n = j, X_{n-1} = i\}$.

Question 3 Find an expression for the Markov renewal functions $m_{ij}(t)$.