IMM - DTU

02407 Stochastic Processes
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Exercise 31

A manufacturing company has a production line consisting of two processing units which are serially connected. Items to be processed arrive according to a Poisson process with intensity λ. The processing time at the first unit is exponentially distributed with mean $\frac{1}{\mu_1}$. The processing time at the second unit is exponentially distributed with mean $\frac{1}{\mu_2}$. It is only possible to queue items at one position in the processing line due to space and cost constraints, either before the first unit or between the first and second unit. In the former case it is not possible to perform an operation on both machines at the same time, thus at least one of the units has to be idle at any given time. In the latter case requests (or items) has to be cancelled if arriving while the first machine is working.

Question 1

Give conditions for stability under the two scenarios.

We will now examine the first scenario in more detail.

Question 2

Derive the mean queue length under this (first) scenario.

Question 3

What is the mean length of a period where the machines (one of them) are working.

We will now consider the the second case for comparison.

Question 4

What is the probability that an arriving request will have to wait.

Question 5

Derive - possibly as a transform expression - the waiting time distribution of an arriving request.

Question 6

Compare the two scenarios.