

Exercise 10 (16/12/91 ex.1)

In connection with an examination of the reliability of some software intended for use in control of modern ferries one is interested in examining a stochastic model of the use of a control program.

The control program works as "state machine" i.e. it can be in a number of different levels, 4 are considered here. The levels depend on the physical state of the ferry. With every shift of time unit during while the program is run, the program will change from level j to level k with probability p_{jk} .

Two possibilities are considered:

- The program has no errors and will run continuously shifting between the four levels.
- The program has a critical error . In this case it is possible:
 - that the error is found, this happens with probability $q_i, i = 1, 2, 3, 4$ depending on the level. The error will be corrected immediately and the program will from then on be without faults.
 - Alternatively the program can stop with a critical error (the ferry will continue to sail, but without control. This happens with probability $r_i, i = 1, 2, 3, 4$.

In general $q_i + r_i < 1$, a program with errors can thus work and the error is not necessarily discovered. It is assumed that detection of an error, as well as the appearance of a fault happens coincidentally with shift between levels.

The program starts running in level 1, and it is known that the program contains one critical error.

Question 1

Formulate a stochastic process (Markov chain) in discrete time describing this system.

Question 2

Characterise the states in the Markov chain.

Vi now consider the case where the stability of the system has been assured, i.e. the error has been found and corrected, and the program has been running for long time without errors. The parameters are as follows.

$$P_{i,i+1} = 0.6 \quad i = 1, 2, 3$$

$$P_{i,i-1} = 0.2 \quad i = 2, 3, 4$$

$$P_{i,j} = 0 \quad |i - j| > 1$$

$$q_i = 10^{-3i}$$

$$r_i = 10^{-3i-5}$$

Question 3

Characterise the stochastic process, that describes the stable system.

Question 4

For what fraction of time will the system be in level 1.