

Exercise 11 (16/12/91 ex.2)

An opinion poll institute has derived a distribution, expressed by the density $f(t) = 0.85 \cdot \frac{1}{2}e^{-\frac{t}{2}} + 0.15 \cdot \frac{1}{6}e^{-\frac{t}{6}}$ describing the time interval of employment for a CEO in a manufacturing company. The time unit is a year. Now consider the case where a CEO has just been engaged in such a company

Question 1

Determine the expected number of CEOs, that will be hired by the company during the next 10 years.

Question 2

Give an expression for the distribution of the time until the third CEO will be hired. The expression can be expressed either as a cumulative distribution function, a density, as a Laplace transform, or as a representation of phase type distribution.

The reason for engaging a new CEO can be that the previous is laid off which happens with probability $p = 0.15$. In this case the company has to reimburse a compensation of 150.000 kr. to the fired CEO. Alternatively the CEO leaves the company for a better offer and he will have to pay a fine to the company of DKKR 75.000 kr. In both cases the company will have expenses of DKKR. 50.000 due to the change in management.

Question 3

Determine the mean and variance of the company's expenditures related to change of CEOs during a 10-year period. It can be stated that the variance of the number of CEO changes is 4.85.

By a closer examination there is some relation between how long time a CEO stays with a company and the reason for his leave. The time describing the employment time for a CEO who decides to leave the company is exponentially distributed with mean 2 years, while the behaviour of a CEO getting fired can be described as an exponentially distribution with mean 6 years. There is no dependence between the reason for leave of two consecutive CEOs. We define the following two types of events

- 1 A CEO leaves the company due to a better offer.
- 2 A CEO is laid off.

In the following $X(t)$ takes the value 1, if the next thing to happen after time t , is that a CEO leaves due to a better offer, (i.e. the next event is of type 1), and 2, if the next thing to happen is a CEO getting laid off. We want to get the probabilities $P_{ij}(t)$:

$$P_{ij}(t) = P\{X(t) = j | \text{At time 0 an event of type } i \text{ occurred}\}$$

Question 4

Present on matrix form the Laplace transform of the matrix $\mathbf{P}(t) = \{P_{ij}(t)\}$. It is sufficient to do this in a way such that the expressions involved is given by the parameters of the system.