IMM - DTU

02407 Stochastic Processes
2001-9-24
NHH/bfn

Exercise 28 (5/1/81 ex.1)

A company delivers goods in lots of N units. the buyers accept that a certain (small) percentage of defective units among the N units delivered, and to assure that the wanted quality is maintained a test sample of n units is examined for control. If there is a maximum of m defective units the lot is accepted if not it is rejected. In what follows we will assume that the N units in a lot is taken from a running production, where the incidence of defective units can be described by a Bernouilliprocess and $\pi =$ the probality that a unit is defect.

Question 1

Find $p =$ the probality that a lot is accepted.

Initially we have assumed, that all lots are controlled in the same way, but it can be appropiate to use a procedure with 2 control levels: normal and tight control. Normal as well as tight control is performed according to the above mentioned procedure, but with tightened control the number of units in the test sample is greater. It will be seen that the control of a lot can be characterized by n= number of units in the test sample and m=maximal number of defective units allowed. We shall assume that with normal control $(n, m) = (n_0, m_0)$ and with tight control $(n, m) = (n_1, m_1)$. The corresponding probabiliteies for acceptance of a lot are p_0 and p_1.

Many rules are possible for change between normal and tight control, but we shall concentrate on the following rule: If the last k lots have been accepted by the control the next lot wil be subjected to normal control. If not the next lot will be subjected to tightened control.

Under these assumptions shift between normal and tightened control can be described by a Markov chain with k+1 states where state nr. l ($\in \{0, 1, \ldots, k\}$) is defined by: the last l lots have been accepted by the control.

Question 2

Give the transition diagram and the matrix of transition probabilities for this Markov chain.

Question 3

Find the probability that a lot is accepted, it is assummed, that a great number of lots have been controlled previously.

The firm has the policy 1) that all units found defective are rejected and replaced by units, that are known with certainty to be in order, and 2) that all units in a rejected lot are examined and if nescessary replaced. When control of a lot is finished we will instead of the n units that constituted the sample have n units, that are known with certainty to be in order.

If the lot was accepted there may be defective units among the $N - n$ units, not taken in the test sample, but if the lot was rejected we will instead of these $N - n$ units have $N - n$ units known with certainty to be in order.

Question 4

How many defective units will on average be found in a great number of controlled lots.

Test sampling is not without cost, it takes time to kontrol a unit so it will be of interest to know haw many units have to be controlled in connection with the acceptance procedure as well as in connection with the replacement of the units that have been found defective.

Question 5

Give the distribution $l - k =$ the number of units, that have to be controlled in order to find k units known with certainty to be in in order, it is assumed that the nescessary are taken from the running production.

Question 6

How many units will in average have to be controlled per lot.