IMM - DTU

04141 Stochastic Processes
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BFN/bfn

Exercise 22 (8/1/98 ex.2)

Packets from two different sources arrive at a communication system. Packets arrive to a communication system from two different sources. One of the sources generates a relatively smooth packet stream, i.e. packets seperated by independent Erlang-2 distributed intervals with mean $\frac{2}{\lambda}$. The other source generates a more bursty stream of packets. For this source the intervals between successive packets can be described as independent hyperexponentially distributed intervals, i.e. a distribution with density function $f(t) = p\mu_1 e^{-\mu_1 t} + (1-p)\mu_2 e^{-\mu_2 t}$.

Question 1

Give a MAP model for each of the two sources describing the process of packet generation.

Question 2

Give a MAP model describing the comined stream of packets.

Question 3

In this question we will consider a general result for Markov chains in continouos time. Consider two Markov chains with generators $\mathbf{A}_i \qquad i = 1, 2$ and stationary probability vectors $\vec{p}_i \qquad i = 1, 2$. Show that the Markov chain with generator $\mathbf{A}_1 \oplus \mathbf{A}_2$ has stationary probability vector $\vec{p}_1 \otimes \vec{p}_2$.
Hint: use formula 1.2 from the note on phase type distributions.

Question 4

Determine the stationary probability vector $\vec{\theta}$ for the underlying Markov of chain of the MAP of question 2.

Question 5

Determine the fundamental rate for the new process (i.e. the fundamental rate of the combined process, packets from either source 1 or 2).
Hint: it might be helpful to use the result of question 3.

Question 6

Give the expected number total number of packets in the interval]0;t] in the stationary case.

For a certain application it is not sufficient to analyze the number of packets arriving in a time interval.

Question 7

For both sources the number of bytes in each packet can be described with a distribution of discrete phase type with mean 104 and variance 350^2. Determine mean and variance of the total number of bytes generated in the interval $]0;t]$. It is known that the variance of the number of packets in the interval $]0;t]$ is known to be $\sigma_p^2(t)$.