IMM - DTU

04141 Stochastic Processes
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BFN/bfn

Exercise 20

We will consider a queueing system of the form $M/M/c/c$. That is a system with arrivals according to a Poisson process with intensity λ and c servers each with exponential holding time with mean $\frac{1}{\mu}$. Arrivals are blocked when all servers are busy, the system has no waiting space.

Question 1

For $c = 2$ formulate the MAP describing the process of completed services.

Question 2

Determine the rate of service completions by calculating the fundamental rate of the MAP. Interpret the result. Is there an easier way to determine the fundamental rate.

Question 3

For $c = 1$ determine the mean and the variance of the time between two departures.

Question 4

Write an expression to determine the covariance of successive intervals in the process of departures for $c = 2$.

We will now consider the process of blocked arrivals.

Question 5

For $c = 3$ formulate the MAP describing the process of blocked arrivals. Characterize the MAP describing blocked arrivals.

Question 6

Determine the covariance of successive intervals in the process of blocked arrivals for $c = 3$.

The system analyzed is a special case of a Birth and Death process.

Question 7

Formulate the MAP describing successive downward transitions in a Birth and Death process.

There is a still more general construction called a Quasi Birth and Death process, with generator matrix $\mathbf{Q}$ given by e.g.

$$\mathbf{Q} = \begin{bmatrix} \mathbf{B}_0 & \mathbf{A}_{0,0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{1,2} & \mathbf{A}_{1,1} & \mathbf{A}_{1,0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{2,2} & \mathbf{A}_{2,1} & \mathbf{A}_{2,0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{3,2} & \mathbf{A}_{3,1} & \mathbf{A}_{3,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{4,2} & \mathbf{A}_{4,1} & \mathbf{A}_{4,0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{5,2} & \mathbf{A}_{5,1} \end{bmatrix}$$

The matrix is partitioned corresponding to levels, i.e. subsets of states.

Question 8

Formulate a MAP describing successive level downward transitions.