IMM - DTU

04141 Stokastiske Processer
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NHH/bfn

Exercise 30 (8/1/94 opg.1)

Let us assume that some kind of tehcnical equipment is needed in order to perform some work Unfortunately this equipment is less reliable than one would wish. Accidents occur during working. The equipment needs repair due to these accidents before it can be used again. According to this it is assumed that

F1A: The occurrence of accidents can be described by a Poisson process wtih intensity λ.

F1B: On the average one accident will occur during the period needed to complete the work

Question 1

Give the probability that the work can be completed without accidents.

Because it is important that the work can be completed without interuptions another unit of the technical equipment has been purchased as a reserve. We will introduce a stochastic process PA in order to describe the system

$$AP : \{X_t, t \in [0, \infty)\}; X_t \in \{1, 2, 3\}$$

$X_t = 1$**:** work is going on at time t and there is a usable unit in reserve.

$X_t = 2$**:** work is going on at time t and there is not a usable unit in reserve.

$X_t = 1$**:** work has stopped.

AP can be characterised by the following assumptions

F2A: Work is commenced when time equals zero and $X_0 = 1$.

F2B: IF an accident occurs at time t AND $X_t = 1$ work continues without significant interruptions and repair on the unit failed is initiated.

F2C: IF an accident occurs at time t AND $X_t = 2$ THEN work stops immeideatly.

In principle when stopped work could be resumed whenever a failed unit becomes available. However, we will not deal with this possibility. The main interest here is whether work can be completed without interuptions and for that reason we further assume

F2D: IF $X_t = 3$ THEN $X_s = 3$ for all $s \geq t$.

Finally it is assumed that

F3A: The duration of a repair is exponentially distributed with mean value μ^{-1}.

F3B: There is a probability at 0.9 that a repair can be completed before the next accident occurs.

Question 2

Find λ and μ assuming that work on average lasts one time unit.

Question 3

It follows from the assumptions that AP is a Markov process. Give the corresponding infinitesimal generator matrix.

Question 4

What is the probability that work can be completed without interuption ?