

## Exercise 27 (4/1/96 ex.1)

A certain type of work requires a special type of equipment. A repairman who runs a business doing this kind of work has acquired the necessary equipment.

We will assume that

1. If the repairman is unoccupied when a customer asks him to do a job, he will start working on the job immediately.
2. If a customer asks him to do a job and he is occupied, he will enter the customer in his order book if the book is empty. If the book already contains an order he will not accept the new customer.
3. The repairman will be idle whenever he completes a job and he has no pending orders.
4. If the order book is not empty, when he finishes a job, he will immediately start working for the customer in the book and remove this customer from the order book.
5. Requests for work arrive in a way that can be described by a Poisson process with intensity 1 pr. time unit.
6. The duration of a job is exponentially distributed with mean  $1/4$  time unit.

### Question 1

What is the fraction of job requests which will be rejected because the repairman is working and the order book is not empty ?

### Question 2

What is the fraction of time where the equipment is used ?

In order to improve the utilization of his equipment the repairman considers renting out the equipment, when not using it. Thus, he will receive rent, but he must take into consideration that he will not be able to work, when the equipment is rented. We will not discuss the profitability but only consider the consequences of the following policy.

1. A rental request will be accepted if and only if the equipment is vacant when the request arrives.

In order to evaluate the consequences of this policy we assume that: 2. The duration of a rental period is exponentially distributed with mean  $1/4$  time unit

1. Arrival of rental requests can be described by a Poisson process with intensity 1 pr. time unit.
2. The duration of a rental period is exponentially distributed with mean  $\frac{1}{4}$  time units.

This rental policy can be described as a Markov process with five states numbered according to the following scheme.

$$\text{State-number} = \text{IB}(1 + \text{UL} + 2 \text{ON})$$

IB, UL, OB are variables that can take values 0 or 1 in accordance with the following

**IB=1** if and only if the equipment is in use

**UL=1** if and only if the equipment is rented

**OB=1** if and only if the repairman has a customer in his order book.

### Question 3

Give the transition probabilities for this Markov process.

### Question 4

Find the stationary distribution for the Markov process

### Question 5

What is the overall utilization of the equipment be, (including rental periods).

### Question 6

What fraction of the requests from customers be rejected caused by the equipment not being available.