Exercise 2

Continuous phasenode distribution

A minor manufacturer of electronic equipment has purchased an even smaller ditto and has thus acquired two pieces of equipment of different types. Both units have a lifetime distribution which can be adequately described by a distribution of phase type. The representation for the lifetime of the first is \((\vec{\alpha}, T)\) while the representation of the other is \((\vec{\beta}, S)\). Further

\[
T = \begin{bmatrix} -\lambda & \lambda \\ 0 & -\lambda \end{bmatrix} \quad \vec{\alpha} = (1, 0)
\]

with \(\lambda = 1\), and:

\[
S = \begin{bmatrix} -\lambda_1 & 0 \\ 0 & -\lambda_2 \end{bmatrix} \quad \vec{\beta} = (p_1, p_2)
\]

with \(\lambda_1 = 3, \lambda_2 = \frac{1}{17}, p_1 = 0.9\). The time unit of the various \(\lambda\) parameters is \((\text{year})^{-1}\). The company wants to decide whether to keep the two units or replace them with equipment. It can be assumed that the time of purchase is arbitrarily chosen in the lifetime distribution.

Question 1 Determine the remaining (or residual) lifetime distribution of the first unit.

Question 2 The probability that the unit survives the first year should exceed 60%. Calculate this probability and hereby determine the decision to be made.

Question 3 Redo question 1 and 2 for the second unit.

Question 4 Finally it is of importance to know the probability that a new unit lasts more than one year. Determine this probability for both types and compare the results from the previous questions. Comment on the results.