IMM - DTU

02407 Stochastic Processes
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BFN/bfn

Exercise 19 (2/1/95 ex.3)

Frequently a model denoted as IPP (Interrupted Poisson Process) is applied as a model for point processes, more irregular than the Poisson process. The process is a two-state continuous time Markov chain, filtrering a Poisson process with intensity λ. The sojourn time in the ON state is exponentially distributed with mean $\frac{1}{\omega_1}$, while the sojourn time in the OFF staet is exsponentially distributed with mean value $\frac{1}{\omega_2}$. The filtrering implies that arrivals in the Poisson process while the state variabel X(t) of the Markov chain has the value X(t)=ON is registrered or accepted while arrivals while X(t)=OFF is canceled (deleted.

Question 1

What is the long term fraction of time with the Markov chain in the ON state.

Question 2

What is the long run average number of arrivals pr. tume unit.

Question 3

Present an expression for the marginal distribution of the time between two consecutive arrivals. Is the process a renewal process ?

We will now consider a superposition of two independent Interrupted Poisson Processes. Both processes are assumed to be stationary, that is the distribution of X(t) does not depend on t.

Question 4

What is the probability the both processes will be in the ON state simultaneously.

We will next consider a superposition of N such processes.

Question 5

What is the probability that exactly i of these processes vill be in the ON state simultaneously.

For one of these processes we can express Var(N(t)), where N(T) is the number of arrivals in the time interval [0,t[by the expression:

$$\mathrm{Var}(N(t)) = \frac{\lambda\omega_2 t}{\omega_1+\omega_2} + \frac{2\lambda^2\omega_1\omega_2 t}{(\omega_1+\omega_2)^3}\left(1 - \frac{1}{(\omega_1+\omega_2)t}\left(1-e^{-(\omega_1+\omega_2)t}\right)\right)$$

Question 6

Determine the relation $\frac{\mathrm{Var}(N(t))}{\mathrm{E}(N(t))}$ for the superposition of the N processes.