Exercise 19 (2/1/95 ex.3)

Frequently a model denoted as IPP (Interrupted Poisson Process) is applied as a model for point processes, more irregular than the Poisson process. The process is a two-state continuous time Markov chain, filtering a Poisson process with intensity $\lambda$. The sojourn time in the ON state is exponentially distributed with mean $\frac{1}{\omega_1}$, while the sojourn time in the OFF state is exponentially distributed with mean value $\frac{1}{\omega_2}$. The filtering implies that arrivals in the Poisson process while the state variable $X(t)$ of the Markov chain has the value $X(t)=\text{ON}$ is registered or accepted while arrivals while $X(t)=\text{OFF}$ is canceled (deleted).

**Question 1**

What is the long term fraction of time with the Markov chain in the ON state.

**Question 2**

What is the long run average number of arrivals pr. time unit.

**Question 3**

Present an expression for the marginal distribution of the time between two consecutive arrivals. Is the process a renewal process ?

We will now consider a superposition of two independent Interrupted Poisson Processes. Both processes are assumed to be stationary, that is the distribution of $X(t)$ does not depend on $t$.

**Question 4**

What is the probability the both processes will be in the ON state simultaneously.
We will next consider a superposition of $N$ such processes.

**Question 5**

What is the probability that exactly $i$ of these processes will be in the ON state simultaneously.

For one of these processes we can express $\text{Var}(N(t))$, where $N(T)$ is the number of arrivals in the time interval $[0,t]$ by the expression:

$$\text{Var}(N(t)) = \frac{\lambda \omega_2 t}{\omega_1 + \omega_2} + \frac{2\lambda^2 \omega_1 \omega_2 t}{(\omega_1 + \omega_2)^3} \left( 1 - \frac{1}{(\omega_1 + \omega_2)t} \left( 1 - e^{-(\omega_1 + \omega_2)t} \right) \right)$$

**Question 6**

Determine the relation $\frac{\text{Var}(N(t))}{E(N(t))}$ for the superposition of the $N$ processes.