IMM - DTU

002407 Stochastic Processes
2020-10-20
BFN/bfn

Exercise 14 (9/1/93 ex.1)

In this exercise we will consider the life time of a special type of electronic equipment. One piece of this equipment will at any given time be in one of 4 possible states

OK The equipment functions intentionally.

Uncertain The equipment has got a minor error and from time to time it malfunctions.

Critical The equipment has a major error implying a serious risk for total failure.

Defect The equipment has broken down.

The sojourn time in each of the 3 states, where the equipment still functions, can be described by an exponential distribution. The mean value of the three distributions are

$$\mu_{ok} = 100 \qquad \mu_{uncertain} = 10 \qquad \mu_{critical} = 1$$

At state change the following will occur

- If the equipment is OK, it can change to be either uncertain or Critical. An OK equipment will become Uncertain with probability, 0.95.
- If the equipment is Uncertain it can become OK (repair), with probability 0.8, or Critical.
- If the equipment is critical it can not be repaired and thus can neither change to be OK nor Uncertain.

The time the equipment spends in each state is independent of the next state being entered. It is further assumed that all pieces of equipment is OK when installed.

Question 1

Formulate a model describing the life time.

Question 2

Calculate the probability that the equipment will be more than 2 time units in the critical state.

We will now consider the case where failed equipment is replaced immediately.

Question 3

What is the expected fraction of time where the equipment in use is in state OK.

Question 4

Show that the mean value μ of a continuous time phase type distribution with representation $(\boldsymbol{\alpha}, \mathbf{S})$, where $\boldsymbol{\alpha}\mathbf{e} = 1$ is given by

$$\mu = (\boldsymbol{\pi}\mathbf{s})^{-1}$$

where $\boldsymbol{\pi}$ is a solution of

$$\boldsymbol{\pi}(\mathbf{S} + \mathbf{s}\boldsymbol{\alpha}) = \mathbf{0}, \qquad \pi\mathbf{e} = 1$$

and

$$\mathbf{S}\mathbf{e} + \mathbf{s} = \mathbf{0}$$

(Hint: the matrix $\mathbf{S}$ is non singular).

Question 5

Calculate the mean life time of the equipment.