IMM - DTU

04141 Stochastic Processes
2020-10-20
BFN/bfn

Exercise 13 (16/12/91 opg.4)

It has been established empirically, that the lifetime of an electronic component can be described by the time till absorption in a Markov chain with the following intensity matrix:

$$\mathbf{Q} = \begin{bmatrix} -1 & \frac{9}{10} & \frac{1}{10} \\ \frac{12}{55} & -\frac{12}{55} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The distribution between these states at time 0 is:

$$\mathbf{p(0)} = \left(\frac{3}{4}, \frac{1}{4}, 0\right)$$

In addition it is known that:

$$\begin{bmatrix} -1 & \frac{9}{10} \\ \frac{12}{55} & -\frac{12}{55} \end{bmatrix}^{-1} = \frac{550}{12} \begin{bmatrix} -\frac{12}{55} & -\frac{9}{10} \\ -\frac{12}{55} & -1 \end{bmatrix} = \begin{bmatrix} -10 & -\frac{165}{4} \\ -10 & -\frac{275}{6} \end{bmatrix}$$

Question 1

Estimate the mean and the variance in the lifetime distribution.

Question 2

Give an explicit (scalar) expression for either the distribution function or the frequence function of the lifetime.

If a component is observed to be defect it is immediately exchanged. The Markov jump process describing the lifetime of the components will thus always be in one of the states 1 or 2.

Question 3

Calculate the probability distributions for the states 1 and 2 in $\mathbf{Q}$.

Question 4

Give an algebraic expression or another representation that describes the remaining time to breakdown measured from a random (time) point and estimate the first and the second (non central) moments in this distribution.