Exercise 13 (16/12/91 opg.4)

It has been established empirically, that the lifetime of an electronic component can be described by the time till absorption in a Markov chain with the following intensity matrix:

\[
Q = \begin{bmatrix}
-1 & \frac{9}{10} & \frac{1}{10} \\
\frac{12}{55} & -\frac{12}{55} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The distribution between these states at time 0 is:

\[
p(0) = \left(\frac{3}{4}, \frac{1}{4}, 0\right)
\]

In addition it is known that:

\[
\begin{bmatrix}
-1 & \frac{9}{10} \\
\frac{12}{55} & -\frac{12}{55}
\end{bmatrix}
\begin{bmatrix}
\frac{12}{55} & -\frac{9}{10} \\
-\frac{12}{55} & -1
\end{bmatrix}
= \frac{550}{12}
\begin{bmatrix}
-10 & -\frac{165}{4} \\
-10 & -\frac{275}{6}
\end{bmatrix}
\]

**Question 1**

Estimate the mean and the variance in the lifetime distribution.

**Question 2**

Give an explicit (scalar) expression for either the distribution function or the frequency function of the lifetime.

If a component is observed to be defect it is immediately exchanged. The Markov jump process describing the lifetime of the components will thus always be in one of the states 1 or 2.

**Question 3**

Calculate the probability distributions for the states 1 and 2 in \(Q\).
Question 4

Give an algebraic expression or another representation that describes the remaining time to breakdown measured from a random (time) point and estimate the first and the second (non central) moments in this distribution.