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ſ	X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
	0	0	0	0	$\frac{1}{3}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1	
0	0	$\frac{1}{3}$	0	

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obal	bility
0	0	0	0			$\frac{1}{3}$	
0	1	1	-1			$\frac{1}{6}$	
1	0	1	1			$\frac{1}{3}$	
1	1	2	0		$\frac{1}{6}$		
		$ \begin{array}{r} +X_3 / X_2 \\ \hline 0 \\ \hline 1 \\ 2 \end{array} $	$-X_{3}$	-1 0 1 6 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 1 3 0	

Question b) With $Z_2 = X_2 - X_3$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obał	oility		
0	0	0	0		0 0			$\frac{1}{3}$	
0	1	1	-1		$\frac{1}{6}$				
1	0	1	1			$\frac{\tilde{1}}{3}$			
1	1	2	0		$\frac{1}{6}$				
	X_2	$+X_3 / X_2$	$-X_{3}$	-1	0	1			
		0		0	$\frac{1}{3}$	0			
		1		$\frac{1}{6}$	Ő	$\frac{1}{3}$			
		2		0	$\frac{1}{6}$	0			

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3)$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obability		
0	0	0	0		0			$\frac{1}{3}$
0	1	1	-1			$\frac{1}{6}$		
1	0	1	1		$\frac{1}{3}$			
1	1	2	0		$\frac{1}{6}$			
		$ \begin{array}{r} +X_3 \ / \ X_2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \\ \end{array} $	$-X_{3}$	-1 0 1 6 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 ¹ / ₃ 0		

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3)$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obability
0	0	0	0			$\frac{1}{3}$
0	1	1	-1			$\frac{1}{6}$
1	0	1	1			$\frac{1}{3}$
1	1	2	0		$\frac{1}{6}$	
		$ \begin{array}{r} +X_3 \ / \ X_2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \\ \end{array} $	$-X_{3}$	-1 0 1 6 0	$\begin{array}{c} 0\\ \frac{1}{3}\\ 0\\ \frac{1}{6} \end{array}$	1 0 1 3 0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3}$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obał	oility
0	0	0	0			$\frac{1}{3}$	
0	1	1	-1			$\frac{1}{6}$	
1	0	1	1		$\frac{1}{3}$		
1	1	2	0		$\frac{1}{6}$		
	X_2	$+X_3 / X_2$	$-X_3$	-1	0	1	
		0		0	$\frac{1}{3}$	0	
		1		$\frac{1}{6}$	0	$\frac{1}{3}$	
		2		Ó	1	0	

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

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X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obal	bility		
0	0	0	0			$\frac{1}{3}$			
0	1	1	-1		1 -1			$\frac{1}{6}$	
1	0	1	1		$\frac{1}{3}$				
1	1	2	0		$\frac{1}{6}$				
	$X_2 + X_3 / X_2 - X_3$			-1	0	1			
		0		0	$\frac{1}{3}$	0			
	1			$\frac{1}{6}$	0	$\frac{1}{3}$			
		2		0	$\frac{1}{6}$	0			

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated.

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X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obat	oility
0	0	0	0			$\frac{1}{3}$	
0	1	1	-1			$\frac{1}{6}$	
1	0	1	1			$\frac{1}{3}$	
1	1	2	0		$\frac{1}{6}$		
		$ \begin{array}{r} +X_3 \ / \ X_2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \\ \end{array} $	$-X_{3}$	-1 0 1 6 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 ¹ / ₃ 0	

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$

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X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obabil	ity		
0	0	0	0		0			$\frac{1}{3}$	
0	1	1	-1			$\frac{1}{6}$			
1	0	1	1		$\frac{1}{3}$				
1	1	2	0			$\frac{1}{6}$			
	$\begin{array}{c c} X_2 + X_3 \ / \ X_2 - X_3 \\ \hline 0 \\ 1 \end{array}$			-1 0 1 6	0 <u>1</u> 3 0	$\frac{1}{\frac{1}{3}}$			
		2		Ő	$\frac{1}{6}$	Õ			

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.

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X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obability		
0	0	0 0		0		$\frac{1}{3}$		
0	1	1	-1		1 -1			$\frac{1}{6}$
1	0	1	1		$\frac{1}{3}$			
1	1	2	0		$\frac{1}{6}$			
	$ \begin{array}{c} X_2 + X_3 \ / \ X_2 - X_3 \\ \hline 0 \\ \hline 1 \\ \end{array} $			0		1 0 1		
		2		$\frac{\frac{1}{6}}{0}$	$\frac{1}{6}$	$\frac{\frac{1}{3}}{0}$		

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1Z_2)$

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X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obabilit	y		
0	0	0	0		0			$\frac{1}{3}$	
0	1	1	-1			$\frac{1}{6}$			
1	0	1	1		$\frac{1}{3}$				
1	1	2	0			$\frac{1}{6}$			
		$ \begin{array}{r} +X_3 \ / \ X_2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \\ \end{array} $	$-X_{3}$	-1 0 ¹ / ₆ 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 ¹ / ₃ 0			

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1Z_2) = E(X_2^2) - E(X_3^2)$

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X_2	X_3	$X_2 + X_3$	$X_2 -$	X_3	Pr	obabi	lity
0	0	0	0			$\frac{1}{3}$	
0	1	1	-1			$\frac{1}{6}$	
1	0	1	1			$\frac{1}{3}$	
1	1	2	0			$\frac{1}{6}$	
		$ \begin{array}{r} +X_3 \ / \ X_2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \end{array} $	$-X_{3}$	-1 0 ¹ / ₆ 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 1 3 0	

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$		Probability		
0	0	0	0		$\frac{1}{3}$		
0	1	1	-1		$\frac{1}{6}$		
1	0	1	1		$\frac{1}{3}$		
1	1	2	0		$\frac{1}{6}$		
		$ \begin{array}{r} +X_3 \ / \ X_2 \\ \hline 0 \\ \hline 1 \\ \hline 2 \\ \end{array} $	$-X_{3}$	-1 0 1 6 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 1 3 0	

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$		Probability		
0	0	0	0		$\frac{1}{3}$		
0	1	1	-1		$\frac{1}{6}$		
1	0	1	1		$\frac{1}{3}$		
1	1	2	0		$\frac{1}{6}$		
	$ \begin{array}{r} X_2 + X_3 \ / \ X_2 - X_3 \\ \hline 0 \\ \hline 1 \\ 2 \end{array} $			-1 0 $\frac{1}{6}$ 0	$ \begin{array}{c} 0 \\ \frac{1}{3} \\ 0 \\ \frac{1}{6} \end{array} $	1 0 ¹ / ₃ 0	

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \neq \frac{5}{6}\frac{1}{6}$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$		Probability		
0	0	0	0		$\frac{1}{3}$		
0	1	1	-1		$\frac{1}{6}$		
1	0	1	1		$\frac{1}{3}$		
1	1	2	0		$\frac{1}{6}$		
	$X_2 + X_3 / X_2 - X_3$			-1	0	1	
	0			0	$\frac{1}{3}$	0	
	1			$\frac{1}{6}$	0	$\frac{1}{3}$	
		2		0	$\frac{1}{6}$	0	

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$. Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \neq \frac{5}{6}\frac{1}{6} = E(Z_1)E(Z_2)$