

IMM - DTU

02405 Probability
2004-5-13
BFN/bfn

Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
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0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0

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0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$

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0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3)$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3)$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3}$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated.

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables
 $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$

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Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables

$Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.

$E(Z_1 Z_2)$

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X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.
 $E(Z_1 Z_2) = E(X_2^2) - E(X_3^2)$

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0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.

$$E(Z_1 Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3}$$

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1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
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Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

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$$E(Z_1 Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

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0	1	1	-1	$\frac{1}{6}$
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$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
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Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables

$Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.

$$E(Z_1 Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \neq \frac{5}{6}$$

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0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3 / X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated.

$$E(Z_1 Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \neq \frac{5}{6} \frac{1}{6} = E(Z_1)E(Z_2)$$