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02405 Probability  
2003-11-13  
BFN/bfn

Question a) By the definition of the c.d.f.

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$$F(x)$$

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$$F(x) = P(X \leq x)$$

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$$F(x) = P(X \leq x) = P(X \leq x|A)$$

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$$F(x) = P(X \leq x) = P(X \leq x|A)P(A)$$

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$$F(x) = P(X \leq x) = P(X \leq x|A)P(A) + P(X \leq x|A^c)P(A^c)$$

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$$F(x) = P(X \leq x) = P(X \leq x|A)P(A) + P(X \leq x|A^c)P(A^c)$$

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$$\frac{dF(x)}{dx} = p \cdot f_1(x) + (1 - p) \cdot f_2(x)$$

with  $f_i(x) = \frac{dF_i(x)}{dx}$ ,  $i = 1, 2$ . Now (see e.g. page 261 top)

$$E(X) = \int x \cdot f(x)dx = \int x(p \cdot f_1(x) + (1 - p) \cdot f_2(x))dx$$

using the linearity of the integral we get

$$= p \int x \cdot f_1(x)dx + (1 - p) \int x \cdot f_2(x)dx = p \cdot E(X_1) + (1 - p) \cdot E(X_2)$$

Question c) We first note that we can derive  $E(X^2)$  in a similar way, thus

$$E(X^2) = p \cdot E(X_1^2) + (1 - p) \cdot E(X_2^2) = p \cdot (Var(X_1) + E(X_1)^2) + (1 - p) \cdot (Var(X_2) + E(X_2)^2)$$

where we have used the computational formula for the variance e.g. page 261. Applying this formula once more we get

$$Var(X) = E(X^2) - E(X)^2$$

$$= p \cdot (Var(X_1) + E(X_1)^2) + (1 - p) \cdot (Var(X_2) + E(X_2)^2) - (p \cdot E(X_1) + (1 - p) \cdot E(X_2))^2$$

Question a) By the definition of the c.d.f. page 311

$$F(x) = P(X \leq x) = P(X \leq x|A)P(A) + P(X \leq x|A^c)P(A^c)$$

using the rule of averaged conditional probabilities page 41. Now introduce the parameters of the exercise ( $P(X \leq x|A) = F_1(x)$  etc.) to get

$$F(x) = p \cdot F_1(x) + (1 - p)F_2(x)$$

Question b) We first find the density of  $X$  assuming  $X_1$  and  $X_2$  continuous (similar calculations can be made in full generality)

$$\frac{dF(x)}{dx} = p \cdot f_1(x) + (1 - p) \cdot f_2(x)$$

with  $f_i(x) = \frac{dF_i(x)}{dx}$ ,  $i = 1, 2$ . Now (see e.g. page 261 top)

$$E(X) = \int x \cdot f(x)dx = \int x(p \cdot f_1(x) + (1 - p) \cdot f_2(x))dx$$

using the linearity of the integral we get

$$= p \int x \cdot f_1(x)dx + (1 - p) \int x \cdot f_2(x)dx = p \cdot E(X_1) + (1 - p) \cdot E(X_2)$$

Question c) We first note that we can derive  $E(X^2)$  in a similar way, thus

$$E(X^2) = p \cdot E(X_1^2) + (1 - p) \cdot E(X_2^2) = p \cdot (Var(X_1) + E(X_1)^2) + (1 - p) \cdot (Var(X_2) + E(X_2)^2)$$

where we have used the computational formula for the variance e.g. page 261. Applying this formula once more we get

$$Var(X) = E(X^2) - E(X)^2$$

$$= p \cdot (Var(X_1) + E(X_1)^2) + (1 - p) \cdot (Var(X_2) + E(X_2)^2) - (p \cdot E(X_1) + (1 - p) \cdot E(X_2))^2$$

$$= p \cdot Var(X_1) + (1 - p) \cdot Var(X_2) + p(1 - p)(E(X_1) - E(X_2))^2$$