IMM - DTU

02405 Probability
2003-11-19
BFN/bfn

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we see that the N_i's are independent Poisson variables.