IMM - DTU

02405 Probability
2004-4-17
BFN/bfn

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$$P(X_2 \leq 60) = \Phi\left(\frac{60 + \frac{1}{2} - 150 \cdot \frac{1}{2}}{\frac{1}{2}\sqrt{150}}\right) = \Phi\left(\frac{-29}{\sqrt{150}}\right)$$

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