Question a) $X$ is binomially distributed
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$. 

Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0)$$
Question a) $X$ is binomially distributed $b \left( 3, \frac{1}{2} \right)$.

\[
P(X = 0) = \frac{1}{8}.
\]
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1)$$
Question a) \( X \) is binomially distributed \( b(3, \frac{1}{2}) \).

\[ P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}. \]
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$P(X = 0) = \frac{1}{8}, \; P(X = 1) = \frac{3}{8}, \; P(X = 2)$$
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables $Z_x$. 

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IMM - DTU

2003-11-1

BFN/bfn

02405 Probability
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

\[ P(X = 0) = \frac{1}{8}, \ P(X = 1) = \frac{3}{8}, \ P(X = 2) = \frac{3}{8}, \ P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables $Z_x$ with binomial distribution
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$. 
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b(3 - x, \frac{1}{2})$.

We can write
Question a) $X$ is binomially distributed $b \left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b \left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$.
Question a) \( X \) is binomially distributed \( b(3, \frac{1}{2}) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b(3 - x, \frac{1}{2}) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \).
Question a) \( X \) is binomially distributed \( b\left(3, \frac{1}{2}\right) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b\left(3 - x, \frac{1}{2}\right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$. We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

\[ P(Y = 0|X = 0) \]
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$. We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0)$$
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).

\[
P(X = 0) = \frac{1}{8}, \ P(X = 1) = \frac{3}{8}, \ P(X = 2) = \frac{3}{8}, \ P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b \left( 3 - x, \frac{1}{2} \right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0 | X = 0) = \frac{1}{8}, \ P(Y = 1 | X = 0) = \frac{3}{8}, \ P(Y = 2 | X = 0) = \frac{3}{8}, \ P(Y = 3 | X = 0) = \frac{1}{8}
\]
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b(3 - x, \frac{1}{2})$. We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0 | X = 0) = \frac{1}{8}, P(Y = 1 | X = 0) = \frac{3}{8}, P(Y = 2 | X = 0) = \frac{3}{8}, P(Y = 3 | X = 0) = \frac{1}{8}$$

For $x = 1$ we get
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1)$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

\[ P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8} \]

For $x = 1$ we get

\[ P(Y = 1|X = 1) = \frac{1}{4} \]
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0 | X = 0) = \frac{1}{8}, P(Y = 1 | X = 0) = \frac{3}{8}, P(Y = 2 | X = 0) = \frac{3}{8}, P(Y = 3 | X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1 | X = 1) = \frac{1}{4}, P(Y = 2 | X = 1)$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}.$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

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For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

\[ P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8} \]

For $x = 1$ we get

\[ P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4} \]

For $x = 2$ we get
Question a) \( X \) is binomially distributed \( b(3, \frac{1}{2}) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b(3 - x, \frac{1}{2}) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2|X = 2)
\]
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b \left( 3 - x, \frac{1}{2} \right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[ P(Y = 0 | X = 0) = \frac{1}{8}, P(Y = 1 | X = 0) = \frac{3}{8}, P(Y = 2 | X = 0) = \frac{3}{8}, P(Y = 3 | X = 0) = \frac{1}{8} \]

For \( x = 1 \) we get

\[ P(Y = 1 | X = 1) = \frac{1}{4}, P(Y = 2 | X = 1) = \frac{1}{2}, P(Y = 3 | X = 1) = \frac{1}{4} \]

For \( x = 2 \) we get

\[ P(Y = 2 | X = 2) = \frac{1}{2} \]
Question a) \( X \) is binomially distributed \( b(3, \frac{1}{2}) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b(3 - x, \frac{1}{2}) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
\]
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).

\[
P(X = 0) = \frac{1}{8}, \ P(X = 1) = \frac{3}{8}, \ P(X = 2) = \frac{3}{8}, \ P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b \left( 3 - x, \frac{1}{2} \right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, \ P(Y = 1|X = 0) = \frac{3}{8}, \ P(Y = 2|X = 0) = \frac{3}{8}, \ P(Y = 3|X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, \ P(Y = 2|X = 1) = \frac{1}{2}, \ P(Y = 3|X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, \ P(Y = 3|X = 2) = \frac{1}{2}
\]

For \( x = 3 \) we get
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) =$$
Question a) $X$ is binomially distributed $b \left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b \left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).

\[
P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}, \quad P(X = 2) = \frac{3}{8}, \quad P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b \left( 3 - x, \frac{1}{2} \right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, \quad P(Y = 1|X = 0) = \frac{3}{8}, \quad P(Y = 2|X = 0) = \frac{3}{8}, \quad P(Y = 3|X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, \quad P(Y = 2|X = 1) = \frac{1}{2}, \quad P(Y = 3|X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, \quad P(Y = 3|X = 2) = \frac{1}{2}
\]

For \( x = 3 \) we get

\[
P(Y = 3|X = 3) = 1
\]

Question c) We find \( P(X = x, Y = y) \)
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b(3 - x, \frac{1}{2})$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. 

Question d) We find the distribution of $Y$ from the distribution table in the previous question.

Question e) Using $P(X = x|Y = y) = P(X = x,Y = y)P(Y = y)/P(Y = y)$ we get for $y = 0$ $P(X = 0|Y = 0) = 1$ for $y = 1$ $P(X = 0|Y = 1) = \frac{1}{3}$, $P(X = 1|Y = 1) = \frac{2}{3}$, $P(X = 2|Y = 1) = \frac{1}{3}$ for $y = 2$ $P(X = 0|Y = 2) = \frac{1}{9}$, $P(X = 1|Y = 2) = \frac{4}{9}$, $P(X = 2|Y = 2) = \frac{4}{9}$ for $y = 3$ $P(X = 0|Y = 3) = \frac{1}{27}$, $P(X = 1|Y = 3) = \frac{2}{9}$, $P(X = 2|Y = 3) = \frac{4}{9}$, $P(X = 3|Y = 3) = \frac{8}{27}$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$. We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

<table>
<thead>
<tr>
<th>$Y$</th>
<th>$X = 0$</th>
<th>$X = 1$</th>
<th>$X = 2$</th>
<th>$X = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>$\frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3-x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

\[
P(Y = 0 | X = 0) = \frac{1}{8}, P(Y = 1 | X = 0) = \frac{3}{8}, P(Y = 2 | X = 0) = \frac{3}{8}, P(Y = 3 | X = 0) = \frac{1}{8}
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For $x = 1$ we get

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\]

For $x = 2$ we get

\[
P(Y = 2 | X = 2) = \frac{1}{2}, P(Y = 3 | X = 2) = \frac{1}{2}
\]

For $x = 3$ we get

\[
P(Y = 3 | X = 3) = 1
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Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y | X = x)$. The distribution table is

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Question f) Best guess $\hat{X}_y$ of $X | Y = y$
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).
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P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}, \quad P(X = 2) = \frac{3}{8}, \quad P(X = 3) = \frac{1}{8}
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For \( x = 2 \) we get
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P(Y = 2|X = 2) = \frac{1}{2}, \quad P(Y = 3|X = 2) = \frac{1}{2}
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For \( x = 3 \) we get
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X/Y & 0 & 1 & 2 & 3 \\
0 & \frac{1}{64} & & & \\
\end{array}
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Question d) We find the distribution of $Y$ from the distribution table in the previous question.

$P(Y = 0) = \frac{1}{64}$, $P(Y = 1) = \frac{9}{64}$, $P(Y = 2) = \frac{27}{64}$, $P(Y = 3) = \frac{27}{64}$

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

\[ P(X = 0|Y = 0) = 1 \]

For $y = 1$

\[ P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3} \]

For $y = 2$

\[ P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9} \]

For $y = 3$

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1 & 0 & \frac{1}{32} & \frac{1}{16} & \frac{3}{32}
\end{array}
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Question f) Best guess \(\hat{X}_y\) of \(X\|Y = y\).
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Question d) We find the distribution of $Y$ from the distribution table in the previous question

\[ P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64} \]

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for $y = 2$

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Question d) We find the distribution of $Y$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$. 

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For \( x = 2 \) we get

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For $x = 2$ we get

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</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

\[
P(Y = 0)
\]
Question a) \( X \) is binomially distributed \( b(3, \frac{1}{2}) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b(3 - x, \frac{1}{2}) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
\]

For \( x = 3 \) we get

\[
P(Y = 3|X = 3) = 1
\]

Question c) We find \( P(X = x, Y = y) = P(X = x)P(Y = y|X = x) \). The distribution table is

\[
\begin{array}{cccc}
X/Y & 0 & 1 & 2 & 3 \\
0 & \frac{1}{64} & \frac{3}{64} & \frac{3}{64} & \frac{1}{64} \\
1 & 0 & \frac{3}{64} & \frac{3}{64} & \frac{3}{64} \\
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3 & 0 & 0 & 0 & \frac{1}{8}
\end{array}
\]

Question d) We find the distribution of \( Y \) from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64},
\]
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

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For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

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Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

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Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1)$$
Question a) \(X\) is binomially distributed \(b\left(3, \frac{1}{2}\right)\).
\[
P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}, \quad P(X = 2) = \frac{3}{8}, \quad P(X = 3) = \frac{1}{8}
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Question b) We introduce the random variables \(Z_x\) with binomial distribution \(b\left(3 - x, \frac{1}{2}\right)\).
We can write \(Y = x + Z_x\) for the conditional distribution of \(Y\). For \(x = 0\) we get
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P(Y = 0|X = 0) = \frac{1}{8}, \quad P(Y = 1|X = 0) = \frac{3}{8}, \quad P(Y = 2|X = 0) = \frac{3}{8}, \quad P(Y = 3|X = 0) = \frac{1}{8}
\]
For \(x = 1\) we get
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\]
For \(x = 2\) we get
\[
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\]

Question d) We find the distribution of \(Y\) from the distribution table in the previous question.
\[
P(Y = 0) = \frac{1}{64}, \quad P(Y = 1) = \frac{9}{64}
\]
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[
P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}, \quad P(X = 2) = \frac{3}{8}, \quad P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

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P(Y = 0|X = 0) = \frac{1}{8}, \quad P(Y = 1|X = 0) = \frac{3}{8}, \quad P(Y = 2|X = 0) = \frac{3}{8}, \quad P(Y = 3|X = 0) = \frac{1}{8}
\]

For $x = 1$ we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, \quad P(Y = 2|X = 1) = \frac{1}{2}, \quad P(Y = 3|X = 1) = \frac{1}{4}
\]

For $x = 2$ we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, \quad P(Y = 3|X = 2) = \frac{1}{2}
\]

For $x = 3$ we get

\[
P(Y = 3|X = 3) = 1
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Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

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Question d) We find the distribution of $Y$ from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64}, \quad P(Y = 1) = \frac{9}{64}, \quad P(Y = 2)
\]
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

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For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

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Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
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For $x = 1$ we get

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P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
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Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$
P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}
$$

Question e) Using $P(X = x|Y = y)$
Question a) \( X \) is binomially distributed \( b\left(3, \frac{1}{2}\right) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
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Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b\left(3 - x, \frac{1}{2}\right) \).

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For \( x = 1 \) we get

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\]

Question d) We find the distribution of \( Y \) from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}
\]

Question e) Using \( P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} \)
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

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\[ P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8} \]

For $x = 1$ we get

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Question d) We find the distribution of $Y$ from the distribution table in the previous question

\[ P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64} \]

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

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For $x = 2$ we get

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Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}$$

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$$P(X = 0|Y = 0)$$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

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</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, \quad P(Y = 1) = \frac{9}{64}, \quad P(Y = 2) = \frac{27}{64}, \quad P(Y = 3) = \frac{27}{64}$$

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

$$P(X = 0|Y = 0) = 1$$

for $y = 1$

$$P(X = 0|Y = 1) = \frac{1}{3}, \quad P(X = 1|Y = 1) = \frac{2}{3}$$

for $y = 2$

$$P(X = 0|Y = 2) = \frac{1}{9}$$
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b(3 - x, \frac{1}{2})$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is:

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Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

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For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

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Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}$$

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for $y = 2$

$$P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}$$
Question a) \( X \) is binomially distributed \( b\left(3, \frac{1}{2}\right) \).

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\]

For \( x = 1 \) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
\]

For \( x = 3 \) we get

\[
P(Y = 3|X = 3) = 1
\]

Question c) We find \( P(X = x, Y = y) = P(X = x) P(Y = y|X = x) \). The distribution table is

| \( X \) | \( 0 \) | \( 1 \) | \( 2 \) | \( 3 \) |
|---|---|---|---|
| 0  | 1/64 | 1/64 | 1/64 | 1/64 |
| 1  | 0    | 3/64 | 3/64 | 1/64 |
| 2  | 0    | 0    | 1/16 | 3/16 |
| 3  | 0    | 0    | 0    | 1/8  |

Question d) We find the distribution of \( Y \) from the distribution table in the previous question

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P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}
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\]

for \( y = 2 \)

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P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}
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for $y = 3$

$$P(X = 0|Y = 3)$$
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

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\]

For $x = 2$ we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
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P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3}
\]

for $y = 2$

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P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}
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for $y = 3$

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\begin{array}{c|cccc}
X/Y & 0 & 1 & 2 & 3 \\
\hline
0 & \frac{1}{64} & \frac{3}{64} & \frac{3}{64} & \frac{1}{64} \\
1 & 0 & \frac{3}{16} & \frac{3}{16} & \frac{3}{16} \\
2 & 0 & 0 & \frac{15}{16} & \frac{15}{16} \\
3 & 0 & 0 & 0 & \frac{1}{8}
\end{array}
\]

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\]

for \( y = 2 \)

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P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}
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for \( y = 3 \)

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For \(x = 2\) we get

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P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
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For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$

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  1 & 0 & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} \\
  2 & 0 & 0 & \frac{1}{16} & \frac{1}{16} \\
  3 & 0 & 0 & 0 & \frac{1}{8}
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for $y = 3$

$$P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27}$$
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Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
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<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>3/64</td>
<td>3/64</td>
<td>1/64</td>
</tr>
<tr>
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<td>1/16</td>
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</tr>
<tr>
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<td>3/16</td>
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<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}$$

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

$$P(X = 0|Y = 0) = 1$$

for $y = 1$

$$P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3}$$

for $y = 2$

$$P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}$$

for $y = 3$

$$P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27}$$

Question f) Best guess $\hat{X}_y$ of $X|Y = y$
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

\[
\begin{align*}
P(X = 0) &= \frac{1}{8},& P(X = 1) &= \frac{3}{8},& P(X = 2) &= \frac{3}{8},& P(X = 3) &= \frac{1}{8}
\end{align*}
\]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}
\]

For $x = 1$ we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
\]

For $x = 2$ we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
\]

For $x = 3$ we get

\[
P(Y = 3|X = 3) = 1
\]

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y = 0$</th>
<th>$Y = 1$</th>
<th>$Y = 2$</th>
<th>$Y = 3$</th>
</tr>
</thead>
<tbody>
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<td>(\frac{3}{64})</td>
<td>(\frac{1}{64})</td>
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<td>0</td>
<td>(\frac{1}{16})</td>
<td>(\frac{3}{16})</td>
<td>(\frac{3}{16})</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>(\frac{1}{8})</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}
\]

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

\[
P(X = 0|Y = 0) = 1
\]

for $y = 1$

\[
P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3}
\]

for $y = 2$

\[
P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}
\]

for $y = 3$

\[
P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27}
\]

Question f) Best guess $\hat{X}_y$ of $X|Y = y$

<table>
<thead>
<tr>
<th>$Y = y$</th>
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<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{X}_y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Question a) \( X \) is binomially distributed \( b(3, \frac{1}{2}) \).
\[
P(X = 0) = \frac{1}{8}, \quad P(X = 1) = \frac{3}{8}, \quad P(X = 2) = \frac{3}{8}, \quad P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b(3 - x, \frac{1}{2}) \).
We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get
\[
P(Y = 0|X = 0) = \frac{1}{8}, \quad P(Y = 1|X = 0) = \frac{3}{8}, \quad P(Y = 2|X = 0) = \frac{3}{8}, \quad P(Y = 3|X = 0) = \frac{1}{8}
\]
For \( x = 1 \) we get
\[
P(Y = 1|X = 1) = \frac{1}{4}, \quad P(Y = 2|X = 1) = \frac{1}{2}, \quad P(Y = 3|X = 1) = \frac{1}{4}
\]
For \( x = 2 \) we get
\[
P(Y = 2|X = 2) = \frac{1}{2}, \quad P(Y = 3|X = 2) = \frac{1}{2}
\]
For \( x = 3 \) we get
\[
P(Y = 3|X = 3) = 1
\]

Question c) We find \( P(X = x, Y = y) = P(X = x)P(Y = y|X = x) \). The distribution table is
\[
\begin{array}{cccc}
X/Y & 0 & 1 & 2 & 3 \\
0 & \frac{1}{64} & \frac{3}{64} & \frac{3}{64} & \frac{1}{64} \\
1 & 0 & \frac{1}{16} & \frac{3}{16} & \frac{1}{16} \\
2 & 0 & 0 & \frac{1}{8} & \frac{1}{8} \\
3 & 0 & 0 & 0 & \frac{1}{8}
\end{array}
\]

Question d) We find the distribution of \( Y \) from the distribution table in the previous question
\[
P(Y = 0) = \frac{1}{64}, \quad P(Y = 1) = \frac{9}{64}, \quad P(Y = 2) = \frac{27}{64}, \quad P(Y = 3) = \frac{27}{64}
\]

Question e) Using \( P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} \) we get for \( y = 0 \)
\[
P(X = 0|Y = 0) = 1
\]
for \( y = 1 \)
\[
P(X = 0|Y = 1) = \frac{1}{3}, \quad P(X = 1|Y = 1) = \frac{2}{3}
\]
for \( y = 2 \)
\[
P(X = 0|Y = 2) = \frac{1}{9}, \quad P(X = 1|Y = 2) = \frac{4}{9}, \quad P(X = 2|Y = 2) = \frac{4}{9}
\]
for \( y = 3 \)
\[
P(X = 0|Y = 3) = \frac{1}{27}, \quad P(X = 1|Y = 3) = \frac{2}{9}, \quad P(X = 2|Y = 3) = \frac{4}{9}, \quad P(X = 3|Y = 3) = \frac{8}{27}
\]

Question f) Best guess \( \hat{X}_y \) of \( X|Y = y \)
\[
\begin{array}{cccc}
Y = y & 0 & 1 & 2 & 3 \\
\hat{X}_y & 0 & 1 & 2 & 3
\end{array}
\]
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b \left( 3 - x, \frac{1}{2} \right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0 | X = 0) = \frac{1}{8}, P(Y = 1 | X = 0) = \frac{3}{8}, P(Y = 2 | X = 0) = \frac{3}{8}, P(Y = 3 | X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1 | X = 1) = \frac{1}{4}, P(Y = 2 | X = 1) = \frac{1}{2}, P(Y = 3 | X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2 | X = 2) = \frac{1}{2}, P(Y = 3 | X = 2) = \frac{1}{2}
\]

For \( x = 3 \) we get

\[
P(Y = 3 | X = 3) = 1
\]

Question c) We find \( P(X = x, Y = y) = P(X = x) P(Y = y | X = x) \). The distribution table is

<table>
<thead>
<tr>
<th>X/Y</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
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<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of \( Y \) from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}
\]

Question e) Using \( P(X = x | Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} \) we get for \( y = 0 \)

\[
P(X = 0 | Y = 0) = 1
\]

for \( y = 1 \)

\[
P(X = 0 | Y = 1) = \frac{1}{3}, P(X = 1 | Y = 1) = \frac{2}{3}
\]

for \( y = 2 \)

\[
P(X = 0 | Y = 2) = \frac{1}{9}, P(X = 1 | Y = 2) = \frac{4}{9}, P(X = 2 | Y = 2) = \frac{4}{9}
\]

for \( y = 3 \)

\[
P(X = 0 | Y = 3) = \frac{1}{27}, P(X = 1 | Y = 3) = \frac{2}{9}, P(X = 2 | Y = 3) = \frac{4}{9}, P(X = 3 | Y = 3) = \frac{8}{27}
\]

Question f) Best guess \( \hat{X}_y \) of \( X | Y = y \)

\[
Y = y \quad 0 \quad 1 \quad 2 \quad 3
\]

\[
\hat{X}_y \quad 0 \quad 1
\]
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

\[ P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8} \]

Question b) We introduce the random variables $Z_x$ with binomial distribution $b(3 - x, \frac{1}{2})$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

\[ P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8} \]

For $x = 1$ we get

\[ P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4} \]

For $x = 2$ we get

\[ P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2} \]

For $x = 3$ we get

\[ P(Y = 3|X = 3) = 1 \]

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

<table>
<thead>
<tr>
<th>X/Y</th>
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<th>3</th>
</tr>
</thead>
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<td>$\frac{3}{64}$</td>
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<td>0</td>
<td>$\frac{1}{16}$</td>
<td>$\frac{3}{16}$</td>
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<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
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<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

\[ P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64} \]

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

\[ P(X = 0|Y = 0) = 1 \]

for $y = 1$

\[ P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3} \]

for $y = 2$

\[ P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9} \]

for $y = 3$

\[ P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27} \]

Question f) Best guess $\hat{X}_y$ of $X|Y = y$

\[ Y = y \quad 0 \quad 1 \quad 2 \quad 3 \\
\hat{X}_y \quad 0 \quad 1 \quad 1 \quad 0 \text{ or } 2 \]
Question a) \( X \) is binomially distributed \( b \left( 3, \frac{1}{2} \right) \).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \( Z_x \) with binomial distribution \( b \left( 3 - x, \frac{1}{2} \right) \).

We can write \( Y = x + Z_x \) for the conditional distribution of \( Y \). For \( x = 0 \) we get

\[
P(Y = 0 | X = 0) = \frac{1}{8}, \quad P(Y = 1 | X = 0) = \frac{3}{8}, \quad P(Y = 2 | X = 0) = \frac{3}{8}, \quad P(Y = 3 | X = 0) = \frac{1}{8}
\]

For \( x = 1 \) we get

\[
P(Y = 1 | X = 1) = \frac{1}{4}, \quad P(Y = 2 | X = 1) = \frac{1}{2}, \quad P(Y = 3 | X = 1) = \frac{1}{4}
\]

For \( x = 2 \) we get

\[
P(Y = 2 | X = 2) = \frac{1}{2}, \quad P(Y = 3 | X = 2) = \frac{1}{2}
\]

For \( x = 3 \) we get

\[
P(Y = 3 | X = 3) = 1
\]

Question c) We find \( P(X = x, Y = y) = P(X = x)P(Y = y | X = x) \). The distribution table is

<table>
<thead>
<tr>
<th>X/Y</th>
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<td>0/8</td>
<td>0/8</td>
<td>0/8</td>
<td>0/8</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of \( Y \) from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64}, \quad P(Y = 1) = \frac{9}{64}, \quad P(Y = 2) = \frac{27}{64}, \quad P(Y = 3) = \frac{27}{64}
\]

Question e) Using \( P(X = x | Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} \) we get for \( y = 0 \)

\[
P(X = 0 | Y = 0) = 1
\]

for \( y = 1 \)

\[
P(X = 0 | Y = 1) = \frac{1}{3}, \quad P(X = 1 | Y = 1) = \frac{2}{3}
\]

for \( y = 2 \)

\[
P(X = 0 | Y = 2) = \frac{1}{9}, \quad P(X = 1 | Y = 2) = \frac{4}{9}, \quad P(X = 2 | Y = 2) = \frac{4}{9}
\]

for \( y = 3 \)

\[
P(X = 0 | Y = 3) = \frac{1}{27}, \quad P(X = 1 | Y = 3) = \frac{2}{9}, \quad P(X = 2 | Y = 3) = \frac{4}{9}, \quad P(X = 3 | Y = 3) = \frac{8}{27}
\]

Question f) Best guess \( \hat{X}_y \) of \( X | Y = y \)

<table>
<thead>
<tr>
<th>( Y = y )</th>
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<th>1</th>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{X}_y )</td>
<td>0</td>
<td>1</td>
<td>1 or 2</td>
<td>2</td>
</tr>
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</table>
Question a) \(X\) is binomially distributed \(b\left(3, \frac{1}{2}\right)\).

\[
P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}
\]

Question b) We introduce the random variables \(Z_x\) with binomial distribution \(b\left(3 - x, \frac{1}{2}\right)\).

We can write \(Y = x + Z_x\) for the conditional distribution of \(Y\). For \(x = 0\) we get

\[
P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}
\]

For \(x = 1\) we get

\[
P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}
\]

For \(x = 2\) we get

\[
P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}
\]

For \(x = 3\) we get

\[
P(Y = 3|X = 3) = 1
\]

Question c) We find \(P(X = x, Y = y) = P(X = x)P(Y = y|X = x)\). The distribution table is

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<td>(\frac{3}{64})</td>
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<td>0</td>
<td>(\frac{1}{16})</td>
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<td>(\frac{3}{16})</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>(\frac{1}{8})</td>
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</tbody>
</table>

Question d) We find the distribution of \(Y\) from the distribution table in the previous question

\[
P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}
\]

Question e) Using \(P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}\) we get for \(y = 0\)

\[
P(X = 0|Y = 0) = 1
\]

for \(y = 1\)

\[
P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3}
\]

for \(y = 2\)

\[
P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}
\]

for \(y = 3\)

\[
P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27}
\]

Question f) Best guess \(\hat{X}_y\) of \(X|Y = y\)

\[
\begin{align*}
Y &= 0 & 1 & 2 & 3 \\
\hat{X}_y &= 0 & 1 & \text{or } 2 & 2
\end{align*}
\]

Question g)
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

$$P(X = 0) = \frac{1}{8}, P(X = 1) = \frac{3}{8}, P(X = 2) = \frac{3}{8}, P(X = 3) = \frac{1}{8}$$

Question b) We introduce the random variables $Z_x$ with binomial distribution $b\left(3 - x, \frac{1}{2}\right)$.

We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

$$P(Y = 0|X = 0) = \frac{1}{8}, P(Y = 1|X = 0) = \frac{3}{8}, P(Y = 2|X = 0) = \frac{3}{8}, P(Y = 3|X = 0) = \frac{1}{8}$$

For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

$$P(Y = 3|X = 3) = 1$$

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{64}$</td>
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<td>$\frac{3}{64}$</td>
<td>$\frac{1}{64}$</td>
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<td>0</td>
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<td>$\frac{3}{32}$</td>
<td>$\frac{3}{32}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{16}$</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}$$

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

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for $y = 1$

$$P(X = 0|Y = 1) = \frac{1}{3}, P(X = 1|Y = 1) = \frac{2}{3}$$

for $y = 2$

$$P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}$$

for $y = 3$

$$P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27}$$

Question f) Best guess $\hat{X}_y$ of $X|Y = y$

$Y = y$ 0 1 2 3

$\hat{X}_y$ 0 1 or 2 2

Question g)
Question a) $X$ is binomially distributed $b\left(3, \frac{1}{2}\right)$.

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For $x = 1$ we get

$$P(Y = 1|X = 1) = \frac{1}{4}, P(Y = 2|X = 1) = \frac{1}{2}, P(Y = 3|X = 1) = \frac{1}{4}$$

For $x = 2$ we get

$$P(Y = 2|X = 2) = \frac{1}{2}, P(Y = 3|X = 2) = \frac{1}{2}$$

For $x = 3$ we get

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<table>
<thead>
<tr>
<th>$X/Y$</th>
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<th>3</th>
</tr>
</thead>
<tbody>
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<tr>
<td>1</td>
<td>0</td>
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<td>$\frac{3}{16}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

Question d) We find the distribution of $Y$ from the distribution table in the previous question

$$P(Y = 0) = \frac{1}{64}, P(Y = 1) = \frac{9}{64}, P(Y = 2) = \frac{27}{64}, P(Y = 3) = \frac{27}{64}$$

Question e) Using $P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$ we get for $y = 0$

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for $y = 1$

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for $y = 2$

$$P(X = 0|Y = 2) = \frac{1}{9}, P(X = 1|Y = 2) = \frac{4}{9}, P(X = 2|Y = 2) = \frac{4}{9}$$

for $y = 3$

$$P(X = 0|Y = 3) = \frac{1}{27}, P(X = 1|Y = 3) = \frac{2}{9}, P(X = 2|Y = 3) = \frac{4}{9}, P(X = 3|Y = 3) = \frac{8}{27}$$

Question f) Best guess $\hat{X}_y$ of $X|Y = y$

<table>
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<tr>
<th>$Y = y$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{X}_y$</td>
<td>0</td>
<td>1</td>
<td>1 or 2</td>
<td>2</td>
</tr>
</tbody>
</table>

Question g)
Question a) $X$ is binomially distributed $b(3, \frac{1}{2})$.

$$\begin{align*}
P(X = 0) &= \frac{1}{8},
P(X = 1) &= \frac{3}{8},
P(X = 2) &= \frac{3}{8},
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\end{align*}$$

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We can write $Y = x + Z_x$ for the conditional distribution of $Y$. For $x = 0$ we get

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P(Y = 3|X = 0) = \frac{1}{8}
\end{align*}$$

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\end{align*}$$

For $x = 2$ we get

$$\begin{align*}
P(Y = 2|X = 2) = \frac{1}{2},
P(Y = 3|X = 2) = \frac{1}{2}
\end{align*}$$

For $x = 3$ we get

$$\begin{align*}
P(Y = 3|X = 3) = 1
\end{align*}$$

Question c) We find $P(X = x, Y = y) = P(X = x)P(Y = y|X = x)$. The distribution table is

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</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$\frac{1}{64}$</td>
<td>$\frac{3}{64}$</td>
<td>$\frac{3}{64}$</td>
<td>$\frac{1}{64}$</td>
</tr>
<tr>
<td>$1$</td>
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<td>$\frac{3}{64}$</td>
<td>$\frac{3}{64}$</td>
</tr>
<tr>
<td>$2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{16}{64}$</td>
<td>$\frac{16}{64}$</td>
</tr>
<tr>
<td>$3$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
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P(X = 0|Y = 0) &= 1
\end{align*}$$

for $y = 1$

$$\begin{align*}
P(X = 0|Y = 1) &= \frac{1}{3},
P(X = 1|Y = 1) = \frac{2}{3}
\end{align*}$$

for $y = 2$

$$\begin{align*}
P(X = 0|Y = 2) = \frac{1}{9},
P(X = 1|Y = 2) = \frac{4}{9},
P(X = 2|Y = 2) = \frac{4}{9}
\end{align*}$$

for $y = 3$

$$\begin{align*}
P(X = 0|Y = 3) = \frac{1}{27},
P(X = 1|Y = 3) = \frac{2}{9},
P(X = 2|Y = 3) = \frac{4}{9},
P(X = 3|Y = 3) = \frac{8}{27}
\end{align*}$$

Question f) Best guess $\hat{X}_y$ of $X|Y = y$

<table>
<thead>
<tr>
<th>$Y = y$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{X}_y$</td>
<td>$0$</td>
<td>$1$</td>
<td>$1$ or $2$</td>
<td>$2$</td>
</tr>
</tbody>
</table>

Question g)