

IMM - DTU

02405 Probability

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EPW/BN

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$$f(t)$$

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$$f(t) = \int_0^t \alpha e^{-\alpha u} \beta e^{-\beta(t-u)} du$$

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$$f(t) = \int_0^t \alpha e^{-\alpha u} \beta e^{-\beta(t-u)} du = \alpha \beta e^{-\beta t} \int_0^t e^{u(\beta-\alpha)} du$$

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Question b) See e.g. page 480 for the means $E(X_i)$ for the exponential variables .

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$$E(Z)$$

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Question b) See e.g. page 480 for the means $E(X_i)$ for the exponential variables .

$$E(Z) = E(X_1) + E(X_2)$$

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Question b) See e.g. page 480 for the means $E(X_i)$ for the exponential variables .

$$E(Z) = E(X_1) + E(X_2) = \frac{1}{\alpha} + \frac{1}{\beta}$$

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Question c) Using the independence of X_1 and X_2 we have

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$$Var(Z)$$

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$$E(Z) = E(X_1) + E(X_2) = \frac{1}{\alpha} + \frac{1}{\beta}$$

Question c) Using the independence of X_1 and X_2 we have

$$Var(Z) = Var(X_1) + Var(X_2)$$

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Question b) See e.g. page 480 for the means $E(X_i)$ for the exponential variables .

$$E(Z) = E(X_1) + E(X_2) = \frac{1}{\alpha} + \frac{1}{\beta}$$

Question c) Using the independence of X_1 and X_2 we have

$$Var(Z) = Var(X_1) + Var(X_2) = \sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}$$

The last equality follows from e.g. page 480.