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$$\frac{1}{z^2 \Gamma(r)\Gamma(s)} \left(\frac{1-z}{z}\right)^{s-1} \int_0^\infty \lambda (\lambda x)^{r+s-1} e^{-\lambda \frac{x}{z}} \mathsf{d}x$$

the function under the integral is very close to a gamma density such that with

$$\frac{1}{z^2 \Gamma(r)\Gamma(s)} \left(\frac{1-z}{z}\right)^{s-1} \Gamma(r+s) z^{r+s-1} \int_0^\infty \lambda \frac{\left(\frac{(\lambda x)}{z}\right)^{r+s-1}}{\Gamma(r+s)} e^{-\lambda \frac{x}{z}} \mathsf{d}x$$

we get the density of a $gamma\left(r+s, \frac{\lambda}{z}\right)$ variable. Thus

$$f_Z(z) = \frac{\Gamma(r+s}{\Gamma(r)\Gamma(s)} (1-z)^{s-1} z^{r+s-(s-1)-2} = \frac{1}{B(r,s)} z^{r-1} (1-z)^{s-1}$$

the density of a $beta(r,s)$ random variable.

We apply the technique of the proof for the distribution of ratios formula page 382-383. Define $Z = \frac{X}{X+Y}$. The event $z < Z < z + \mathrm{d}z$ occurs whenever Y is between the two lines $\frac{x}{x+y} = z + \mathrm{d}z$ and $\frac{x}{x+y} = z$. We get the length of the vertical side of the rectangle by solving for y in the two equations above. Thus

$$y_2 = \left(\frac{1}{z} - 1\right)x, y_1 = \left(\frac{1}{z + \mathrm{d}z} - 1\right)x, \qquad y_2 - y_1 = \frac{x\mathrm{d}z}{z(z + \mathrm{d}z)} = \tilde{=} \frac{x\mathrm{d}z}{z^2}$$

We have derived a general formula for the density of $Z = \frac{X}{X+Y}$ for non negative X and Y

$$\int_0^\infty \frac{x}{z^2} f_X(x) f_Y\left(\frac{(1-z)x}{z}\right) \mathrm{d}x$$

We now insert the gamma densities of X and Y (see page 481) to get

$$\int_0^\infty \frac{x}{z^2} \lambda \frac{(\lambda x)^{r-1}}{\Gamma(r)} e^{-\lambda x} \lambda \frac{\left(\lambda \frac{(1-z)x}{z}\right)^{s-1}}{\Gamma(s)} e^{-\lambda \frac{(1-z)x}{z}} \mathrm{d}x$$

We simplify to get

$$\frac{1}{z^2 \Gamma(r)\Gamma(s)} \left(\frac{1-z}{z}\right)^{s-1} \int_0^\infty \lambda (\lambda x)^{r+s-1} e^{-\lambda \frac{x}{z}} \mathrm{d}x$$

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we get the density of a $gamma\left(r+s, \frac{\lambda}{z}\right)$ variable. Thus

$$f_Z(z) = \frac{\Gamma(r+s}{\Gamma(r)\Gamma(s)} (1-z)^{s-1} z^{r+s-(s-1)-2} = \frac{1}{B(r,s)} z^{r-1}(1-z)^{s-1}$$

the density of a $beta(r,s)$ random variable.

Three lines to follow

1. We see directly from the calculations
2. Considering

$$P\left(z < \frac{X}{X+Y} < z + \mathsf{d}z | w < X + Y < w + \mathsf{d}w\right)$$

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3. Using the division rule page 425