

IMM - DTU

02405 Probability

2005-10-29

BFN/bfn

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Question c) Maximum and minimum (see exercise 4.6.3). Also note that the marginal distributions are those of max and min from page 316/317.