

IMM - DTU

02405 Probability
2005-4-7
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Question a)

$$P(Z = 1) = P(S < T) = P(S \leq T) = P(T \leq S) = P(Z = 0) \Rightarrow \\ P(Z = 1) = P(Z = 0) = \frac{1}{2}$$

Question b) It is intuitively tempting to claim that X and Z are independent. This is an example where intuition is correct. However one should be careful and should be able to verify with rigorous arguments.

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Inserting we get

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A similar argument shows the independence of Y and Z .

Question c) Independence between which variable attains the k 'th order statistic and the value of the k 'th order statistic.

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