IMM - DTU

02405 Probability
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Question c) We apply the inverse distribution function method suggested page 320-323.

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Question c) We apply the inverse distribution function method suggested page 320-323. Thus

$$U = 1 - e^{-\lambda X}$$

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