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$$P(T \leq x) = F_{Wei}(x) = \int_0^x \lambda\alpha u^{\alpha-1}e^{-\lambda u^{\alpha}}\mathrm{d}u = \left[e^{-\lambda u^{\alpha}}\right]_{u=0}^{u=x} = 1 - e^{-\lambda x^{\alpha}}$$

Now inserting $x = t^{\frac{1}{\alpha}}$ we get

$$P(T^{\alpha} \leq t) = 1 - e^{-\lambda\left(t^{\frac{1}{\alpha}}\right)^{\alpha}}$$

Question a) Using the one-to-one change of variable results page 304 we get
$Y = g(T) = T^\alpha$, $T = Y^{\frac{1}{\alpha}}$, $\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha t^{\alpha-1}$

$$f_Y(y) = \lambda\alpha t^{\alpha-1} e^{-\lambda t^\alpha} \frac{1}{\alpha t^{\alpha-1}} = \lambda e^{-\lambda y}$$

the exponential density.

Question b) Once again using the one-to-one change of variable results page 304 we get
$T = g(U) = (-\frac{1}{\lambda}\ln(U))^{\frac{1}{\alpha}}, U = e^{-\lambda T^\alpha}$, $\left|\frac{\mathrm{d}t}{\mathrm{d}u}\right| = \frac{1}{\lambda\alpha}\frac{1}{u}(-\frac{1}{\lambda}\ln(U))^{\frac{1}{\alpha}-1}$

$$f_T(t) = 1\frac{1}{\frac{1}{\lambda\alpha}\frac{1}{u}(-\ln(u))^{\frac{1}{\alpha}-1}} = \lambda\alpha e^{-t^\alpha}(t^\alpha)^{1-\frac{1}{\alpha}} = \lambda\alpha t^{\alpha-1}e^{-t^\alpha}$$

a Weibull(λ, α) density.

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$$P(T^\alpha \leq t) = P(T \leq t^{\frac{1}{\alpha}})$$

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Now inserting $x = t^{\frac{1}{\alpha}}$ we get

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which shows us that T^α has an exponential distribution.

Question b) We examine the random variable $Y = \left(-\lambda^{-1} \ln\left(U\right)\right)^{\frac{1}{\alpha}}$.

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where the last equality follows from page 487 (cumulative distribution function), which was to be shown.