02405 Probability 2003-10-15 BFN/bfn

First we introduce Y

02405 Probability 2003-10-15 BFN/bfn

First we introduce Y = g(U)

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y=g(U)=U^2$ 

IMM - DTU	02405 Probability
	2003-10-15
	BFN/bfn
First we introduce $Y = g(U) = U^2$ and note that $g()$ is strict	tly increasing on $]0,1[$ .

02405 Probability

First we introduce  $Y=g(U)=U^2$  and note that g() is strictly increasing on ]0,1[. We then apply the formula in the box on page 304.

02405 Probability

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0,1[. We then apply the formula in the box on page 304. In our case we have

 $f_X(x)$ 

02405 Probability

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

 $f_X(x) = 1$  for 0 < x < 1,

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0,1[. We then apply the formula in the box on page 304. In our case we have

 $f_X(x) = 1$  for 0 < x < 1,  $y = g(x) = x^2$ ,

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

 $f_X(x) = 1$  for 0 < x < 1,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,

02405 Probability

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1 \text{ for } 0 < x < 1, \qquad y = g(x) = x^2, \qquad x = \sqrt{y}, \qquad \frac{dy}{dx}$$

02405 Probability

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x$ 

02405 Probability

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$
  
$$F_{U^2}(y) = P(U^2 \le y) = P(U \le \sqrt{y}) = \sqrt{y}$$

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

Inserting in the formula

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$
  
$$F_{U^2}(y) = P(U^2 \le y) = P(U \le \sqrt{y}) = \sqrt{y}$$

The last equality follows from the cumulative distribution function (CDF) of a Uniformly distributed random variable (page 487).

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

Inserting in the formula

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$
$$F_{U^2}(y) = P(U^2 \le y) = P(U \le \sqrt{y}) = \sqrt{y}$$

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

Inserting in the formula

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$
$$F_{U^2}(y) = P(U^2 \le y) = P(U \le \sqrt{y}) = \sqrt{y}$$

$$f_{U^2}(y)$$

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

Inserting in the formula

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$
  
$$F_{U^2}(y) = P(U^2 \le y) = P(U \le \sqrt{y}) = \sqrt{y}$$

$$f_{U^2}(y) = \frac{\mathsf{d}F_{U^2}(y)}{\mathsf{d}y}$$

02405 Probability 2003-10-15 BFN/bfn

First we introduce  $Y = g(U) = U^2$  and note that g() is strictly increasing on ]0, 1[. We then apply the formula in the box on page 304. In our case we have

$$f_X(x) = 1$$
 for  $0 < x < 1$ ,  $y = g(x) = x^2$ ,  $x = \sqrt{y}$ ,  $\frac{dy}{dx} = 2x = 2\sqrt{y}$ 

Inserting in the formula

$$f_Y(y) = \frac{1}{2\sqrt{y}} \qquad 0 < y < 1$$
$$F_{U^2}(y) = P(U^2 \le y) = P(U \le \sqrt{y}) = \sqrt{y}$$

$$f_{U^2}(y) = \frac{\mathsf{d}F_{U^2}(y)}{\mathsf{d}y} = \frac{1}{2\sqrt{y}}, 0 < y < 1$$