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02405 Probability
2003-11-12
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This is a special case of the χ^2 distribution, here with 1 degree of freedom. The general case is introduced page 365. The distribution is extremely important in statistics (and probability).

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