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Question a)

$\Gamma(r+1)$

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$$\Gamma(r+1) = \int_0^\infty x^r e^{-x} \mathrm{d}x$$

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$$\Gamma(r+1) = \int_0^\infty x^r e^{-x} \mathsf{d}x = \left[x^r(-e^{-x})\right]_0^\infty - \int_0^\infty r x^{r-1}(-e^{-x}) \mathsf{d}x$$

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$$\Gamma(r+1) = \int_0^\infty x^r e^{-x} \mathrm{d}x = \left[x^r(-e^{-x})\right]_0^\infty - \int_0^\infty r x^{r-1}(-e^{-x}) \mathrm{d}x = r \int_0^\infty r x^{r-1} e^{-x} \mathrm{d}x = r\Gamma(r)$$

Question b) For $r = 1$ we have

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and the result is proved by induction.

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since $E((\lambda T)^n) = n!$ (the variable $Y = \lambda T$ is an $exponential(1)$ distributed random variable).