

IMM - DTU

02405 Probability

2003-10-15

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$$f(x) = 30 \cdot x^2(1-x)^2 \quad 0 < x < 1$$

This is an example of the Beta distribution page 327,328,478.

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$$\int_0^1 x f(x) dx = \int_0^1 x 30 \cdot x^2 \left(\sum_{i=0}^2 \binom{2}{i} (-x)^i \right) dx = 30 \sum_{i=0}^2 \binom{2}{i} (-1)^i \left[\frac{x^{i+4}}{i+4} \right]_{x=0}^{x=1} = \frac{1}{2}$$

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$$SD(X_{3,3})^2$$

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