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We denote the event

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We denote the event that there are 3 sixes

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P(B \mid A)=\underline{P(B \cap A)}
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By the multiplication rule we get the probability $P(B \cap A)=P(A \mid B) P(B)$, thus as a speical case of Bayes Rule page 49 we get

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Now the probability of $P(A)$ is given by

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Now the probability of $P(A)$ is given by the binomial distribution

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3 \\
6^{5} \\
1
\end{array}\right) \frac{5^{2}}{6^{3}}}{\binom{5}{2} \frac{5^{5}}{6^{8}}}
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$$

a hypergeometric probability. The result generalizes. If we have $x$ successes in $n$ trials then the probability of having $y \leq x$ successes in $m \leq n$ trials is given by

$$
\frac{\binom{m}{y}\binom{n-m}{x-y}}{\binom{n}{x}}
$$

The probabilities do not depend on $p$.

