IMM - DTU

02405 Probability
2006-3-9
BFN/bfn

Solution for review exercise 24 (chapter 3) in Pitman

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The sum of $I_{X>Y}+I_{Y>Z}+I_{Z>Z}$ can not be greater than 2, thus the smallest of the three probabilities

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$$\{n, n-1, \ldots, 2, 1\}, \{1, n, n-1, \ldots, 3, 2\}, \ldots, \{n-1, n-2, \ldots, 1, n\}$$

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$$\{n,n-1,\ldots,2,1\},\{1,n,n-1,\ldots,3,2\},\ldots,\{n-1,n-2,\ldots,1,n\}$$

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We can achieve $p = P(X > Y) = P(Y > Z) = P(Z > X)$ for $p_1 = \frac{3-\sqrt{5}}{2}$ and p_2

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