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02405 Probability
2006-11-1
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Solution for review exercise 2.28 (chapter 2) in Pitman

Question a) We define the events

Solution for review exercise 2.28 (chapter 2) in Pitman

Question a) We define the events A_i that

IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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Question a) We define the events A_i that person i receives a correct letter.

IMM - DTU

02405 Probability
2006-11-1
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Question a) We define the events A_i that person i receives a correct letter. Each person has a probability of

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IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

Solution for review exercise 2.28 (chapter 2) in Pitman

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IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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$P(\cup_{i=1}^n A_i)$

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$$P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$$

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$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) - \sum_{i<j} P(A_i \cap A_j)$$

IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots (-1)^{n-1} P\left(\cap_{i=1}^n A_i\right)$$

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$$= \sum_{i=1}^n \binom{n}{i} (-1)^{i-1} \frac{(n-i)!}{n!}$$

IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots (-1)^{n-1} P\left(\cap_{i=1}^n A_i\right)$$

$$= \sum_{i=1}^n \binom{n}{i} (-1)^{i-1} \frac{(n-i)!}{n!} = \sum_{i=1}^n (-1)^{i-1} \frac{1}{i!}$$

IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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$$P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots (-1)^{n-1} P\left(\cap_{i=1}^{n} A_i\right)$$

$$= \sum_{i=1}^{n} \binom{n}{i} (-1)^{i-1} \frac{(n-i)!}{n!} = \sum_{i=1}^{n} (-1)^{i-1} \frac{1}{i!}$$

Question b) The sum in question a) is close to the first n terms of the

IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

Solution for review exercise 2.28 (chapter 2) in Pitman

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$$P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots (-1)^{n-1} P\left(\cap_{i=1}^{n} A_i\right)$$

$$= \sum_{i=1}^{n} \binom{n}{i} (-1)^{i-1} \frac{(n-i)!}{n!} = \sum_{i=1}^{n} (-1)^{i-1} \frac{1}{i!}$$

Question b) The sum in question a) is close to the first n terms of the Taylor expansion of

IMM - DTU

02405 Probability
2006-11-1
BFN/bfn

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$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots (-1)^{n-1} P\left(\cap_{i=1}^n A_i\right)$$

$$= \sum_{i=1}^n \binom{n}{i} (-1)^{i-1} \frac{(n-i)!}{n!} = \sum_{i=1}^n (-1)^{i-1} \frac{1}{i!}$$

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$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots (-1)^{n-1} P\left(\cap_{i=1}^n A_i\right)$$

$$= \sum_{i=1}^n \left(\begin{array}{c} n \\ i \end{array}\right) (-1)^{i-1} \frac{(n-i)!}{n!} = \sum_{i=1}^n (-1)^{i-1} \frac{1}{i!}$$

Question b) The sum in question a) is close to the first n terms of the Taylor expansion of e^{-1}. Thus approximately for large n

Solution for review exercise 2.28 (chapter 2) in Pitman

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$$P(\cup_{i=1}^n A_i) \tilde{=}$$

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Question b) The sum in question a) is close to the first n terms of the Taylor expansion of e^{-1}. Thus approximately for large n

$$P(\cup_{i=1}^n A_i) \cong 1 - \frac{1}{e}$$