IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 8 (chapter 6) in Pitman

Question a) Using the multiplication rule see e.g. page 425 top we get

$$f(x,y) = f_Y(y) f_X(x|Y=y) = 2e^{-2y}\frac{1}{y}e^{-\frac{x}{y}}$$

The marginal density $f_X(x)$ of X is given by

$$f_X(x) = \int_0^\infty 2e^{-2y}\frac{1}{y}e^{-\frac{x}{y}}\mathrm{d}y$$

a non-standard density.

Question b) Using average conditional expectation page 425 bottom we get

$$E(X) = E(E(X|Y)) = E(Y) = \frac{1}{2}$$

noting that the roles of X and Y are interchanged.

Question c) Similarly

$$E(XY) = E(E(XY|Y)) = E(YE(X|Y)) = E(Y^2) = Var(Y) + (E(Y))^2 = \frac{1}{2}$$

We have $E(X^2) = E(E(X^2|Y)) = E(2Y^2) = 1$. Thus $Var(X) = SD(X)^2 = 1 - \frac{1}{4} = \frac{3}{4}$ and $SD(Y) = \frac{1}{2}$. Finally $Corr(X,Y) = \frac{\frac{1}{2}-\frac{1}{4}}{\frac{\sqrt{3}}{2}\frac{1}{2}} = \frac{\sqrt{3}}{3}$