IMM - DTU

02405 Probability
2003-11-1
BFN/bfn

Solution for review exercise 1 (chapter 5) in Pitman

First apply the definition of conditional probability page 36

$$P\left(Y \geq \frac{1}{2} | Y \geq X^2\right) = \frac{P\left(Y \geq \frac{1}{2} \cap Y \geq X^2\right)}{P(Y \geq X^2)}$$

The joint density of X and Y is the product of the marginal densities since X and Y are independent (page 349). We calculate the denominator using the formula for the probability of a set B page 349

$$P(Y \geq X^2) = \int_0^1 \int_{x^2}^1 1 \cdot 1 \cdot \mathrm{d}y\mathrm{d}x = \int_0^1 (1 - x^2)\mathrm{d}x = 1 - \frac{1}{3} = \frac{2}{3}$$

and the numerator

$$P\left(Y \geq \frac{1}{2} \cap Y \geq X^2\right) = P(Y \geq X^2) - P\left(Y < \frac{1}{2} \cap Y \geq X^2\right)$$

Now for the last term

$$P\left(Y < \frac{1}{2} \cap Y \geq X^2\right) = \int_0^{\frac{1}{\sqrt{2}}} \int_{x^2}^{\frac{1}{2}} 1 \cdot \mathrm{d}y\mathrm{d}x = \int_0^{\frac{1}{\sqrt{2}}} (\frac{1}{2} - x^2)\mathrm{d}x$$

$$= \frac{1}{2}\frac{1}{\sqrt{2}} - \frac{1}{3}\frac{1}{2}\frac{1}{\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

Finally we get

$$P\left(Y \geq \frac{1}{2} | Y \geq X^2\right) = \frac{\frac{2}{3} - \frac{1}{3\sqrt{2}}}{\frac{2}{3}} = 1 - \frac{\sqrt{2}}{4}$$