IMM - DTU

02405 Probability
2003-11-1
BFN/bfn

Solution for review exercise 7 (chapter 4) in Pitman

Question a) We require $\int_{\infty}^{\infty} f(x)\mathrm{d}x = \int_{-\infty}^{\infty} \alpha e^{-\beta|x|}\mathrm{d}x = 1$. We have $\alpha = \frac{\beta}{2}$ since $\int_0^{\infty} \beta e^{-\beta x}\mathrm{d}x = 1$.

Question b) We immediately get $E(X) = 0$ since $f(x)$ is symmetric around zero. The second moment $E(X^2)$ is identical to the second moment of the standard exponential, which we can find from the computational formula for the variance. We additionally have $Var(X) = E(X^2)$ since $E(X) = 0$.

$$Var(X) = E(X^2) = \frac{1}{\beta^2} + \left(\frac{1}{\beta}\right)^2 = \frac{2}{\beta^2}$$

Question c)

$$P(|X| > y) = 2P(X > y) = 2\int_y^{\infty} \frac{\beta}{2} e^{-\beta t}\mathrm{d}t = \int_y^{\infty} \beta e^{-\beta t}\mathrm{d}t = e^{-\beta y}$$

the standard exponential survival function.

Question d) From the result in c) we are lead to

$$P(X \leq x) = \begin{cases} \frac{1}{2}e^{\beta x} & x < 0 \\ 0.5 + \frac{1}{2}e^{-\beta x} & 0 < x \end{cases}$$