IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 26 (chapter 4) in Pitman

Question a)

$$E(W_t) = E\left(Xe^{tY}\right) = E(X)E\left(e^{tY}\right)$$

by the independence of X and Y. We find $E\left(e^{tY}\right)$ from the definition of the mean.

$$E\left(e^{tY}\right) = \int_1^{\frac{3}{2}} e^{ty} \cdot 2\mathrm{d}y = \frac{2e^t}{t}\left(e^{\frac{t}{2}} - 1\right)$$

Inserting this result and $E(X) = 2$ we get

$$E(W_t) = 2\frac{2e^t}{t}\left(e^{\frac{t}{2}} - 1\right)$$

Alternatively we could derive the joint density of X and Y to

$$f(x, y) = 2(2x)^3e^{-2x}, \qquad 0 < x, 0 < y < 1$$

where we have used that X has Gamma (4,2) density, and apply the formula for $E(g(X, Y))$ page 349.

Question b) Since X and Y are independent we find $E(W_t^2)$

$$E(W_t^2) = E(X^2)E\left(\left(e^{tY}\right)^2\right)$$

where $E(X^2) = Var(X) + (E(X))^2 = 5$, see eg. page 481. Next we derive

$$E\left(\left(e^{tY}\right)^2\right) = \frac{e^{2t}}{t}\left(e^t - 1\right)$$

and apply the computational formula for the variance page 261

$$SD(W_t) = \sqrt{5\frac{e^{2t}}{t}\left(e^t - 1\right) - \left(2\frac{2e^t}{t}\left(e^{\frac{t}{2}} - 1\right)\right)^2} =$$