IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 25 (chapter 4) in Pitman

Question a) We first note that the range of Y is $0 < Y \leq \frac{1}{2}$.

$$P(Y \leq y) = P\left(U \leq \frac{1}{2}\right) P\left((Y \leq y | U \leq \frac{1}{2}\right) + P\left(\frac{1}{2} < U\right) P\left((Y \leq y | \frac{1}{2} < U\right) = 2P(U \leq y)$$

The density is 2 for $0 < y < \frac{1}{2}$ 0 elsewhere.

Question b) The standard uniform density $f(y) = 1$ for $0 < y < 1$, 0 elsewhere.

Question c)

$$E(Y) = \frac{\frac{1}{2} - 0}{2} = \frac{1}{4}, Var(Y) = \frac{\left(\frac{1}{2} - 0\right)^2}{12} = \frac{1}{48}$$