IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 23 (chapter 4) in Pitman

We introduce $Y = M - 3$ such that Y has the exponential distribution with mean 2.

Question a)

$$E(M) = E(Y+3) = E(Y) + 3 = 5 \qquad Var(M) = Var(Y+3) = Var(Y) = 4$$

where we have used standard rules for mean and variance see eg. page 249, and the result page 279 for the variance of the exponential distribution.

Question b) We get the density $f_M(m)$ of the random variable M is

$$f_M(m) = \frac{1}{2}e^{-\frac{1}{2}(m-3)} \qquad m > 3.$$

from the stated assumptions. We can apply the box page 304 to get

$$f_X(x) = \frac{f_M(m)}{\frac{\mathrm{d}x}{\mathrm{d}m}} = \frac{\frac{1}{2}e^{-\frac{1}{2}(\log(x)-3)}}{x} = \frac{\frac{e^{\frac{3}{2}}}{2}}{x\sqrt{x}}, \qquad x > e^3$$

where $X = g(M) = e^M$. Alternatively

$$F_X(x) = P(X \leq x) = P(\log(X) \leq \log(x) = P(\log(X) - 3 \leq \log(x) - 3)$$

$$= P(Y \leq \log(x) - 3) = 1 - e^{\frac{-(\log(x)-3)}{2}} = 1 - \frac{e^{\frac{3}{2}}}{\sqrt{x}} \qquad x > e^3$$

taking derivative we get

$$f_X(x) = \frac{\mathrm{d}F_X(x)}{\mathrm{d}x} == \frac{\frac{e^{\frac{3}{2}}}{2}}{x\sqrt{x}}, \qquad x > e^3$$

Question c) We do the calculations in terms of the random variables $Y_i = M_i - 3$, $M_i = \log(X_i)$. Here X_i denotes the magnitude of the i'th earthquake. From Example 3 page 317 we know that the minimum Z of the Y_i's, $Z = \min(Y_1, Y_2)$ is exponentially distributed with mean 1.

$$P(M > 4) = P(Z > 1) = e^{-1}$$