IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 21 (chapter 4) in Pitman

Question a) We first note using exercise 4.3.4 page 301 and exercise 4.4.9 page 310 hat R_1 and R_2 are both Weibull$\left(\alpha = 2, \lambda = \frac{1}{2}\right)$ distributed. The survival function is thus (from E4.3.4) $G(x) = e^{-\frac{1}{2}x^2}$. We now apply the result for the minimum of independent random variables page 317 to get

$$P(Y \leq y) = P(\min(R_1, R_2) \leq y) = 1 - P(R_1 > y, R_2 > y) = 1 - P(R_1 > y)(R_2 > y)$$

$$= 1 - e^{-\frac{1}{2}y^2} e^{-\frac{1}{2}y^2} = 1 - e^{-y^2}$$

a new Weibull distribution with $\alpha = 2$ and $\lambda = 1$. If we did not recognize the distribution as a Weibull we would derive the survival function of the R_i's by

$$P(R_i > x) = \int_x^{\infty} u e^{-\frac{1}{2}u^2} \mathrm{d}u = e^{-\frac{1}{2}x^2}$$

We find the density using (5) page 297 or directly using E4.3.4 (i)

$$f_Y(y) = 2y e^{-y^2}$$

Question b) This is a special case of E4.4.9 a). We can re-derive this result using the change of variable formula page 304. With $Z = g(Y) = Y^2$ we get $\frac{\mathrm{d}g(y)}{\mathrm{d}y} = 2y$. Inserting we get

$$f_Z(z) = 2y e^{-y^2} \frac{1}{2y} = e^{-z}$$

an exponential(1) distribution.

Question c) We have $E(Z) = 1$ (see e.g. the mean of an exponential variable page 279 or the distribution summary page 477 or page 480).