IMM - DTU

02405 Probability
2003-10-16
BFN/bfn

Solution for review exercise 13 (chapter 4) in Pitman

We introduce the random variables $N_{\text{loc}}(t)$ and $N_{\text{dis}}(t)$ as the number of local respectively long distance calls arriving within time t (where t is given in minutes).

Question a)

$$P(N_{\text{loc}}(1) = 5, N_{\text{dis}}(1) = 3) = P(N_{\text{loc}}(1) = 5)P(N_{\text{dis}}(1) = 3)$$

due to the independence of the Poisson processes. The variables $N_{\text{loc}}(t)$ and $N_{\text{dis}}(t)$ has Poisson distributions (page 289) such that

$$P(N_{\text{loc}}(1) = 5, N_{\text{dis}}(1) = 3) = \frac{(\lambda_{\text{loc}} \cdot 1)^5}{5!} e^{-\lambda_{\text{loc}} \cdot 1} \cdot \frac{(\lambda_{\text{dis}} \cdot 1)^3}{3!} e^{-\lambda_{\text{dis}} \cdot 1} = \frac{\lambda_{\text{dis}}^3 \lambda_{\text{loc}}^5}{5!3!} e^{-\lambda_{\text{loc}} - \lambda_{\text{dis}}}$$

Question b) The sum of two indpendent Poisson random variables is Poisson distributed (boxed result page 226), leading to

$$P(N_{\text{loc}}(3) + N_{\text{dis}}(3) = 50) = \frac{((\lambda_{\text{loc}} + \lambda_{\text{dis}})3)^{50}}{50!} e^{-(\lambda_{\text{loc}} + \lambda_{\text{dis}})3}$$

Question c) We now introduce the random variables Si_{loc} and Si_{dis} as the time of the i'th local and long distance call respectively. These random variables are Gamma distributed according to the box on the top of page 286 or to 4. page 289 The probability in question can be expressed as The waiting time to the first long distance in terms of calls are geometrically distributed

$$P(X > 10) = (1 - p_{\text{dis}})^{10} = \left(\frac{\lambda_{\text{loc}}}{\lambda_{\text{loc}} + \lambda_{\text{dis}}} \right)^{10}$$