Solution for review exercise 25 (chapter 3) in Pitman

Question a) The joint distribution of (Y_1, Y_2) is given by

Y_1/Y_2	0	1	2
0	$\frac{9}{36}$	$\frac{6}{36}$	$\frac{3}{36}$
1	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$
2	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

as a check we verify that the sum of all entries in the table is 1. We derive the distribution of $Y_1 + Y_2$

$Y_1 + Y_2 = i$	0	1	2	3	4
$P(Y_1 + Y_2 = i)$	$\frac{9}{36}$	$\frac{12}{36}$	$\frac{10}{36}$	$\frac{4}{36}$	$\frac{1}{36}$

Question b)

$$E(3Y_1+2Y_2) = E(3Y_1)+E(2Y_2) = 3E(Y_1)+2E(Y_2) = 5E(Y_1) = 5\left(0\cdot\frac{1}{2}+1\cdot\frac{1}{3}+2\cdot\frac{1}{6}\right) = \frac{10}{3}$$

The first equality is true due to the addition rule for expectations (page 181), the second equality is true due to the result for linear functions of random variables page 175 b., the third equality is true since Y_1 and Y_2 has the same distribution, and the fourth equality is obtained from the definition of the mean see page 181.

Question c)

$$f(x) = \begin{cases} 0 & \text{for } X \leq 3 \\ 1 & \text{for } 4 \leq X \leq 5 \\ 2 & \text{for } X = 6 \end{cases}$$

or something similar.