IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 19 (chapter 3) in Pitman

Question a)

$$P(Y \geq X) = \sum_{x=0}^{\infty} P(X = x)P(Y \geq X|X = x)$$

now X and Y are independent such that

$$P(Y \geq X) = \sum_{x=0}^{\infty} P(X = x)P(Y \geq x)$$

There is a convenient formula for the tail probabilities of a geometric distribution, see eg. page 482. We need to adjust this result to the present case of a geometric distribution with range $0, 1, \ldots$ (counting only failures), such that $P(Y \geq x) = (1 - p)^x$. We now insert this result and the Poisson densities to get

$$P(Y \geq X) = \sum_{x=0}^{\infty} \frac{\mu^x}{x!} e^{-\mu}(1 - p)^x = e^{-\mu} e^{\mu(1-p)} = e^{-\mu p}$$

where we have used the exponential series $\sum_{x=0}^{\infty} \frac{(\mu(1-p))^x}{x!} = e^{\mu(1-p)}$.

Question b)

$$e^{-\mu p} = e^{-\frac{1}{2}} = 0.6065$$