IMM - DTU

02405 Probability
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BFN/bfn

Solution for review exercise 13 (chapter 2) in Pitman

The probability that the manufacturer will have to replace a packet is

$$P(\text{replace}) = \sum_{i=3}^{50} \binom{50}{i} 0.01^i 0.99^{50-i} = 1 - \sum_{i=0}^{2} \binom{50}{i} 0.01^i 0.99^{50-i}$$

$$= 0.99^{50}\left(1 + \frac{0.01}{0.99}\cdot 50\left(1 + \frac{0.01}{0.99}\cdot\frac{49}{2}\right)\right) = 0.0138$$

Pitman claims this probability to be 0.0144. We evaluate the second probability using the Normal approximation to the Binomial distribution. Let X denote the number of packets the manufacturer has to replace. The random variable X follows a Binomial distribution with $n = 4000$ and $p =$. We can evaluate the probability using the normal approximation.

$$P(X > 40) = 1 - P(X \leq 40) \tilde{=} 1 - \Phi\left(\frac{40 + \frac{1}{2} - 4000\cdot 0.0138}{\sqrt{4000\cdot 0.0138\cdot 0.9862}}\right)$$

$$1 - \Phi\left(\frac{-14.77}{7.38}\right) = 1 - \Phi(-2.00) = 0.9772$$

Slightly different from Pitman's result due to the difference above.