IMM - DTU

02405 Probability
2004-5-13
BFN/bfn

Solution for exercise 6.5.6 in Pitman

Question a)

$$P(X > kY) = P(X - kY > 0)$$

From the boxed result page 363 we know that $Z = X - kY$ is $normal(0, 1 + k^2)$ distributed, thus $P(X - kY > 0) = \frac{1}{2}$.

Question b) Arguing along the same lines we find $P(U > kV) = \frac{1}{2}$.

Question c)

$$P(U^2 + V^2 < 1) = P(3X^2 + Y^2 + 2\sqrt{3}XY + X^2 + 3Y^2 - 2\sqrt{3}XY < 1)$$

$$= P\left(X^2 + Y^2 < \frac{1}{4}\right) = 1 - e^{-\frac{1}{8}} = 0.118$$

where we have used $X^2 + Y^2 \in exponential(0.5)$ in the last equality (page 360, 364-366, 485).

Question d)

$$X = v + \sqrt{3}Y \in normal(v, 3)$$