IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 6.5.4 in Pitman

Question a) We have from the boxed result page 363 that $X + 2Y$ is normally distributed with mean $\mu = 0 + 2 \cdot 0 = 0$ and variance $\sigma^2 = 1 + 4 \cdot 1 = 5$. We now evaluate

$$P(X + 2Y \leq 3) = P\left(\frac{X + 2Y}{\sqrt{5}} \leq \frac{3}{\sqrt{5}}\right) = \Phi\left(\frac{3}{\sqrt{5}}\right) = \Phi(1.34) = 0.9099$$

Question b) We have from the boxed result page 451

$$Y = \frac{1}{2}X + \sqrt{1 - \frac{1}{4}}Z$$

where X and Z are indpendent standard normal variables. Thus

$$X + 2Y = 2X + \sqrt{3}Z$$

This is the sum of two independent normal variables which itself is $Normal(0, 2^2 + \sqrt{3}^2)$ distributed. Thus

$$P(X + 2Y \leq 3) = \Phi\left(\frac{3}{\sqrt{7}}\right) = \Phi(1.13) = 0.8708$$