IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 6.5.2 in Pitman

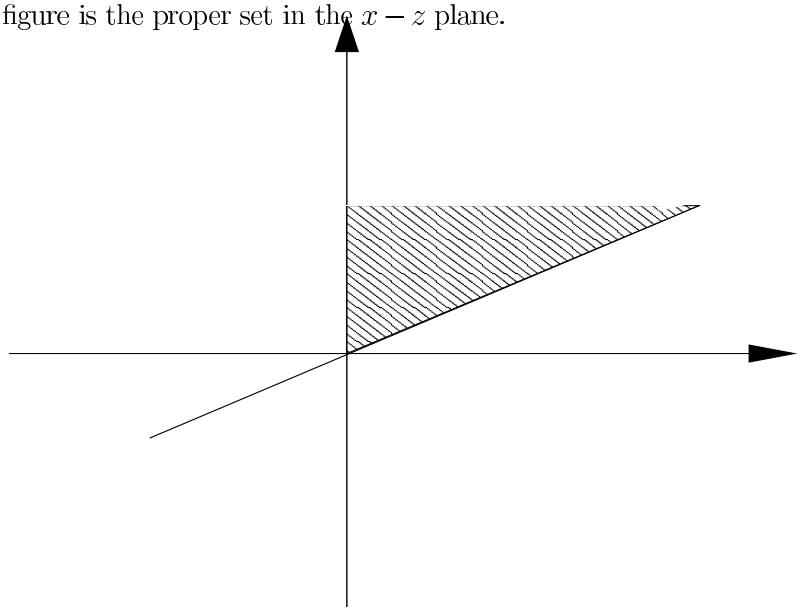
First we realize that we can consider standard normal variates ignoring specific parameter values. For a bivariate standard normal distribution (X, Y) with correlation 0.5 we need to solve for

$$P(Y > X | X > 0) = \frac{P(Y > X > 0)}{P(X > 0)}$$

Now applying the technique of example 2 page 454 we get

$$\frac{P(Y > X > 0)}{P(X > 0)} = \frac{P(\rho \cdot X + \sqrt{1-\rho^2} Z > X > 0)}{P(X > 0)} = \frac{P\left(Z > \frac{1-\rho}{\sqrt{1-\rho^2}} X > 0\right)}{P(X > 0)}$$

where (X, Z) are bivariate normal and independent.The shaded area in the figure is the proper set in the $x - z$ plane.



Using the rotational symmetry we find the probability to be $\frac{60}{180} = \frac{1}{3}$ or $\frac{\frac{60}{360}}{\frac{1}{2}} = \frac{1}{3}$.