Solution for exercise 6.5.1 in Pitman

Question a) We define U to be a students score on the PSAT test and V to be the score of the same student on the PSAT test. The pair (U, V) follows a general bivariate normal distribution as given in the box at the bottom of page 454. The probability in question is $P(V > 1300|U = 1000)$ which we rewrite

$$P(V > 1300|U = 1000) = P\left(\frac{V - 1300}{90}\middle|\frac{U - 1200}{100} = -2\right)$$

Now using the definition on page 454 together with the definition of the standard bivariate normal distribution page 451 we get

$$P(0.6 \cdot X + \sqrt{1 - 0.6^2}Z > 0|X = -2) = P\left(Z > \frac{1.2}{0.8}\right) = 1 - \Phi(1.5) = 0.0668$$

Question b) The solution to this question is closely related to the method of Example 2 page 457. First we realize that we can consider standard normal variates. Using the notation of the previous question we formulate the problem as

$$P(Y > 0|X < 0) = P(0.6 \cdot X + 0.8 \cdot Z > 0|X < 0) = \frac{P(0.6 \cdot X + 0.8 \cdot Z > 0, X < 0)}{P(X < 0)}$$

Now using the rotational symmetry we see that $P(0.6 \cdot X + 0.8 \cdot Z > 0, X < 0) = \frac{90 - \tan^{-1}\left(\frac{3}{4}\right)}{360} = 0.14758$. Finally $P(Y > 0|X < 0) = \frac{0.14758}{0.5} = 0.2952$.

Question c) We formulate the question using the notation of question a) as

$$P(V - U > 50)$$

we get

$$P(1300 + 90 \cdot Y - (1200 + 100 \cdot X) > 50)$$
$$= P(1300 + 90 \cdot (\rho X + \sqrt{1 - \rho^2}Z) - (1200 + 100 \cdot X) > 50)$$
$$= P(72Z - 46X \geq -50) = 1 - \Phi\left(-\frac{50}{\sqrt{46^2 + 72^2}}\right) = \Phi(0.585) = 0.72$$