IMM - DTU

02405 Probability
2004-5-13
BFN/bfn

Solution for exercise 6.4.7 in Pitman

Question a)

X_2	X_3	$X_2 + X_3$	$X_2 - X_3$	Probability
0	0	0	0	$\frac{1}{3}$
0	1	1	-1	$\frac{1}{6}$
1	0	1	1	$\frac{1}{3}$
1	1	2	0	$\frac{1}{6}$

$X_2 + X_3$ / $X_2 - X_3$	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{6}$	0	$\frac{1}{3}$
2	0	$\frac{1}{6}$	0

Question b) With $Z_2 = X_2 - X_3$ we get $E((X_2 - X_3)^3) = E(Z_2^3) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$.

Question c) X_2 and X_3 are independent thus uncorrelated. The new variables $Z_1 = X_2 + X_3$ and $Z_2 = X_2 - X_3$ are correlated. $E(Z_1 Z_2) = E(X_2^2) - E(X_3^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \neq \frac{5}{6}\frac{1}{6} = E(Z_1)E(Z_2)$