IMM - DTU

02405 Probability
2003-11-11
BFN/bfn

Solution for exercise 6.4.6 in Pitman

X and Y are clearly not indpendent.

$P(X = 0|Y = 12) = P(X_1 - X_2 = 0|X_1 + X_2 = 12) = 1 \neq P(X_1 - X_2 = 0) = P(X = 0)$

However, X and Y are uncorrelated:

$$Cov(X, Y) = E((X - E(X))(Y - E(Y))) = E(XY) - E(X)E(Y) = E(XY)$$

$$= E((X_1 - X_2)(X_1 + X_2)) = E(X_1^2 - X_2^2) = E(X_1^2) - E(X_2^2) = 0$$

using the definition of covariance page 630