IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 6.4.2 in Pitman

From $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$ we realize that $P(A)$ is a weighted average of $P(A|B)$ and $P(A|B^c)$, thus one and only one of

1. $P(A|B) = P(A) = P(A|B^c)$
2. $P(A|B) < P(A) < P(A|B^c)$
3. $P(A|B) > P(A) > P(A|B^c)$

is true.

Question a) Obvious from page 42.

Question b) We have

$$Cov(I_A, I_B) = P(A \cap B) - P(A)P(B) = P(A|B)P(B) - P(A)P(B) = (P(A|B) - P(A))P(B) > 0$$

Question c) As for c) interchanging the roles of B and B^c.

Question d) Once again obvious from page 42.

Question e) We have $(P(A|B) - P(A))P(B) > 0$ since A and B are positively dependent. We dedeuce that $P(A|B) > P(A)$ implying $P(A|B) > P(A|B^c)$

Question f) As for e).