IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 6.3.5 in Pitman

We note that Y for given $X = x$ is uniformly distributed, on $1 + x$ for $-1 < x < 0$ and on $1 - x$ for $0 < x < 1$. Thus

$$F(y|x) = P(Y \leq y|X = x) = \frac{y}{1 - |x|}, 0 < y < 1 - |x|$$

Question a) We have $P\left(Y \geq \frac{1}{2}|X = x\right) = 1 - F\left(\frac{1}{2}|x\right)$

Question b) We have $P\left(Y \leq \frac{1}{2}|X = x\right) = F\left(\frac{1}{2}|x\right)$

Question c) Since Y for given $X = x$ is uniformly distributed we can apply results for the uniform distribution, see e.g. the distribution summary page 477 or 487. We get

$$E(Y|X = x) = \frac{1 - |x|}{2}$$

Question c) Similarly

$$Var(Y|X = x) = \frac{(1 - |x|)^2}{12}$$