

Solution for exercise 6.3.13 in Pitman

Question a) We determine $P(X < Y)$ using the law of averaged conditional probabilities

$$\begin{aligned} P(X < Y) &= P(X < Y|Y = 0)P(Y = 0) + P(X < Y|Y = 1)P(Y = 1) \\ &+ P(X < Y|Y = 2)P(Y = 2) + P(X < Y|Y = 3)P(Y = 3) + P(X < Y|Y \geq 4)P(Y \geq 4) \\ &= 1 \cdot e^{-\lambda} + \frac{2}{3}\lambda e^{-\lambda} + \frac{1}{3}\frac{\lambda^2}{2}e^{-\lambda} = e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2} \right) \end{aligned}$$

leading to

$$P(X < Y) = 1 - e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2} \right)$$

Question b)

Question c)