IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 6.2.1 in Pitman

We have the joint distribution (distribution table) (see e.g. exercise 3.1.4)

$X\backslash Y$	1	2	3	4	5	6
1	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
2	0	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
3	0	0	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
4	0	0	0	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$
5	0	0	0	0	$\frac{1}{36}$	$\frac{1}{18}$
6	0	0	0	0	0	$\frac{1}{36}$

We find the conditional distributions to be

$Y:$	1	2	3	4	5	6
$P(X=1\|Y=y)$	1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{2}{11}$
$P(X=2\|Y=y)$	0	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{2}{11}$
$P(X=3\|Y=y)$	0	0	$\frac{1}{5}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{2}{11}$
$P(X=4\|Y=y)$	0	0	0	$\frac{1}{7}$	$\frac{2}{9}$	$\frac{2}{11}$
$P(X=5\|Y=y)$	0	0	0	0	$\frac{1}{9}$	$\frac{2}{11}$
$P(X=6\|Y=y)$	0	0	0	0	1	$\frac{1}{11}$

and

x	$P(Y=1\|X=x)$	$P(Y=1\|X=x)$	$P(Y=1\|X=x)$	$P(Y=1\|X=x)$	$P(Y=1\|X=x)$	$P(Y$
1	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	
2	0	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	
3	0	0	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	
4	0	0	0	$\frac{1}{5}$	$\frac{2}{5}$	
5	0	0	0	0	$\frac{1}{3}$	
6	0	0	0	0	0	

where we have used the short $P(Y=y|x)$ for $P(Y=y|X=x)$.

Such that

$Y:$	1	2	3	4	5	6
$E(X\|Y=y)$	1	$\frac{4}{3}$	$\frac{9}{5}$	$\frac{16}{7}$	$\frac{25}{9}$	$\frac{36}{11}$

and

x	$E(Y\|X=x)$
1	$\frac{41}{11}$
2	$\frac{38}{9}$
3	$\frac{33}{7}$
4	$\frac{26}{5}$
5	$\frac{17}{3}$
6	6