IMM - DTU

02405 Probability
2004-4-17
BFN/bfn

Solution for exercise 6.1.5 in Pitman

Question a) The probability in distribution in question is $P(X_1 = x_1 | X_1 + X_2 = n)$. Using the definition of conditioned probabilities

$$P(X_1 = x_1 | X_1 + X_2 = n) = \frac{P(X_1 = x_1, X_1 + X_2 = n)}{P(X_1 + X_2 = n)}$$

$$= \frac{P(X_1 = x_1, X_2 = n - x_1)}{P(X_1 + X_2 = n)} = \frac{P(X_1 = x_1)P(X_2 = n - x_1)}{P(X_1 + X_2 = n)}$$

where we have used the independence of X_1 and X_2 and the last equality. Now using the Poisson probability expression and the boxed result page 226

$$P(X_1 = x_1 | X_1 + X_2 = n) = \frac{\frac{\lambda_1^{x_1}}{x_1!} e^{-\lambda_1} \frac{\lambda_2^{n-x_1}}{(n-x_1)!} e^{-\lambda_2}}{\frac{(\lambda_1+\lambda_2)^n}{n!} e^{-(\lambda_1+\lambda_2)}}$$

$$= \frac{n!}{x_1!(n-x_1)!} \frac{\lambda_1^{x_1} \lambda_2^{n-x_1}}{(\lambda_1 + \lambda_2)^n} = \binom{n}{x_1} p^{x_1} (1-p)^{n-x_1}$$

with $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$.

Question b) Let X_i denote the number of eggs laid by insect i. The probability in question is $P(X_1 \geq 90) = P(X_2 \leq 60)$. Now $X_i \in \textit{binomial}\left(150, \frac{1}{2}\right)$. With the normal approximation to the binomial distribution page 99 to get

$$P(X_2 \leq 60) = \Phi\left(\frac{60 + \frac{1}{2} - 150 \cdot \frac{1}{2}}{\frac{1}{2}\sqrt{150}}\right) = \Phi\left(\frac{-29}{\sqrt{150}}\right) = \Phi(-2.37) = 0.0089$$