IMM - DTU

02405 Probability
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BFN/bfn

Solution for exercise 5.4.7 in Pitman

Question a) We apply the method used to derive the distribution of ratios page 382. Such that

$$P(z < Z < z + \mathrm{d}z) = \int_x P(x < X < x + \mathrm{d}x, z < XY < z + \mathrm{d}z)$$

Instead of the cone on page 382 we now have an area between the two curves $xy = z$ and $xy = z + \mathrm{d}z$. Thus we have that the area of the parallelogram for fixed x is approximately equal to

$$\mathrm{d}x \left(\frac{z + \mathrm{d}z}{x} - \frac{z}{x} \right) = \frac{\mathrm{d}x\mathrm{d}z}{x}$$

We get the density of Z by integration over x. Thus

$$f_Z(z) = \int_x \frac{1}{x} f\left(x, \frac{z}{x}\right) \mathrm{d}x$$

Question b) This part follows more or less directly from page 372, such that $Z = X - Y$ has density

$$f_Z(z) = \int_x f(x, x - z) \mathrm{d}x$$

Question c) Introduce $W = 2Y$. The density $f_W(w)$ of W is $\frac{1}{2} f_Y(w)$ from the linear change of variable principle; see e.g. page 333. We now apply the general convolution result page 372 or page 386 for the variables X and W to get

$$f_Z(z) = \int_x \frac{1}{2} f\left(x, \frac{z - x}{2}\right) \mathrm{d}x$$