

Solution for exercise 5.4.6 in Pitman

The argument of example 2 page 375 is easily generalized. Since X_i is gamma(r_i, λ) distributed we can write X_i as

$$X_i = \sum_{j=1}^{r_i} W_{ij}$$

where W_{ij} are independent exponential(λ) variables. Thus

$$\sum_{i=1}^n X_i = \sum_{i=1}^n \sum_{j=1}^{r_i} W_{ij}$$

a sum of $\sum_{i=1}^n r_i$ exponential(λ) random variables. The sum is gamma($\sum_{i=1}^n r_i, \lambda$) distributed.